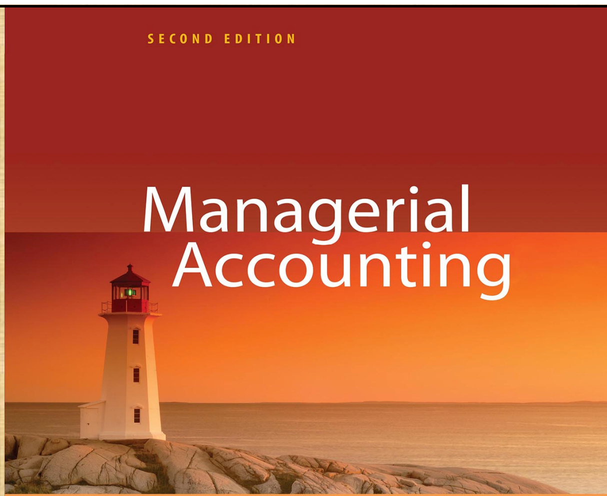

SECOND EDITION

Managerial Accounting

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ASIA GLOBAL EDITION

Differential Analysis: The Key to Decision Making

Chapter 14

Learning Objective 1

Identify relevant and irrelevant costs and benefits in a decision.

Cost Concepts for Decision Making

A **relevant cost** is a cost that differs between alternatives.

Identifying Relevant Costs

An **avoidable cost** is a cost that can be eliminated, in whole or in part, by choosing one alternative over another. Avoidable costs are relevant costs. Unavoidable costs are irrelevant costs.

Two broad categories of costs are never relevant in any decision. They include:

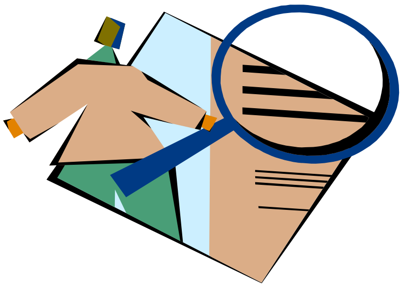
1. **Sunk costs.**
2. **Future costs that do not differ between the alternatives.**

Relevant Cost Analysis: A Two-Step Process

Step 1 Eliminate costs and benefits that do not differ between alternatives.

Step 2 Use the remaining costs and benefits that differ between alternatives in making the decision. The costs that remain are the differential, or avoidable, costs.

Different Costs for Different Purposes



Costs that are relevant in one decision situation may not be relevant in another context. Thus, in each decision situation, the manager must examine the data at hand and isolate the relevant costs.

Identifying Relevant Costs

Cynthia, a Malaysian student studying in Penang, is considering visiting her friend in Kuala Lumpur. She can drive or take the budget airline. By car, it is 230 miles to her friend's apartment. She is trying to decide which alternative is less expensive and has gathered the following information:

		Annual Cost of Fixed Items	Cost per Mile
1	Annual straight-line depreciation on car	$ 2,800	$ 0.280
2	Cost of gasoline		0.100
3	Annual cost of auto insurance and license	1,380	0.138
4	Maintenance and repairs		0.065
5	Parking fees at school	360	0.036
6	Total average cost		$ 0.619

$45 per month x 8 months

$2.70 per gallon ÷ 27 MPG

$24,000 cost – $10,000 salvage value ÷ 5 years

Identifying Relevant Costs

	Automobile Costs (based on 10,000 miles driven per year)	Annual Cost of Fixed Items	Cost per Mile
1	Annual straight-line depreciation on car	$ 2,800	$ 0.280
2	Cost of gasoline		0.100
3	Annual cost of auto insurance and license	1,380	0.138
4	Maintenance and repairs		0.065
5	Parking fees at school	360	0.036
6	Total average cost		$ 0.619

	Some Additional Information	
7	Reduction in resale value of car per mile of wear	$ 0.026
8	Round-tip airfare	$ 104
9	Benefits of relaxing on plane trip	????
10	Cost of putting dog in kennel while gone	$ 40
11	Benefit of having car in Kuala Lumpur	????
12	Hassle of parking car in Kuala Lumpur	????
13	Per day cost of parking car in Kuala Lumpur	$ 25

Identifying Relevant Costs

Which costs and benefits are relevant in Cynthia's decision?

The cost of the car is a sunk cost and is not relevant to the current decision.

The annual cost of insurance is not relevant. It will remain the same if she drives or takes the plane.

However, the cost of gasoline is clearly relevant if she decides to drive. If she takes the plane, the cost would not be incurred, so it varies depending on the decision.

Identifying Relevant Costs

Which costs and benefits are relevant in Cynthia's decision?

The cost of maintenance and repairs is relevant. In the long-run these costs depend upon miles driven.

The monthly school parking fee is not relevant because it must be paid if Cynthia drives or takes the plane.

At this point, we can see that some of the average cost of $0.619 per mile are relevant and others are not.

Identifying Relevant Costs

Which costs and benefits are relevant in Cynthia's decision?

The decline in resale value due to additional miles is a relevant cost.

The round-trip airfare is clearly relevant. If she drives the cost can be avoided.

Relaxing on the plane is relevant even though it is difficult to assign a dollar value to the benefit.

The kennel cost is not relevant because Cynthia will incur the cost if she drives or takes the plane.

Identifying Relevant Costs

Which costs and benefits are relevant in Cynthia's decision?

The cost of parking in Kuala Lumpur is **relevant** because it can be avoided if she takes the plane.

The benefits of having a car in Kuala Lumpur and the problems of finding a parking space are both **relevant** but are difficult to assign a dollar amount.

Identifying Relevant Costs

From a financial standpoint, Cynthia would be better off taking the plane to visit her friend. Some of the non-financial factor may influence her final decision.

Relevant Financial Cost of Driving	
Gasoline (460 @ $0.100 per mile)	$ 46.00
Maintenance (460 @ $0.065 per mile)	29.90
Reduction in resale (460 @ $0.026 per mile)	11.96
Parking in Kuala Lumpur (2 days @ $25 per day)	50.00
Total	$ 137.86

Relevant Financial Cost of Taking the Plane	
Round-trip ticket	$ 104.00

Total and Differential Cost Approaches

The management of a company is considering a new labor saving machine that rents for $3,000 per year. Data about the company's annual sales and costs with and without the new machine are:

	Current Situation	Situation With New Machine	Differential Costs and Benefits
Sales (5,000 units @ $40 per unit)	$ 200,000	$ 200,000	-
Less variable expenses:			
Direct materials (5,000 units @ $14 per unit)	70,000	70,000	-
Direct labor (5,000 units @ $8 and $5 per unit)	40,000	25,000	15,000
Variable overhead (5,000 units @ $2 per unit)	10,000	10,000	-
Total variable expenses	120,000	105,000	-
Contribution margin	80,000	95,000	15,000
Less fixed expense:			
Other	62,000	62,000	-
Rent on new machine	-	3,000	(3,000)
Total fixed expenses	62,000	65,000	(3,000)
Net operating income	$ 18,000	$ 30,000	12,000

Total and Differential Cost Approaches

As you can see, the only costs that differ between the alternatives are the direct labor costs savings and the increase in fixed rental costs.

	Current Situation	Situation With New Machine	Differential Costs and Benefits
Sales (5,000 units @ $40 per unit)	$ 200,000	$ 200,000	-
Less variabl			
Direct ma			-
Direct lab			15,000
Variable			-
Total varia			-
Contributio			15,000
Less fixed			
Other			-
Rent on n			(3,000)
Total fixed			(3,000)
Net operati			12,000

We can efficiently analyze the decision by looking at the different costs and revenues and arrive at the same solution.

Net Advantage to Renting the New Machine	
Decrease in direct labor costs (5,000 units @ $3 per unit)	$ 15,000
Increase in fixed rental expenses	(3,000)
Net annual cost saving from renting the new machine	$ 12,000

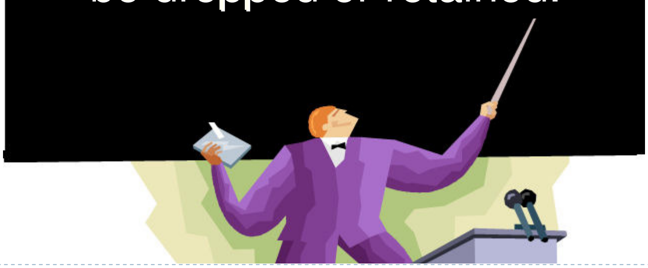
Total and Differential Cost Approaches

Using the differential approach is desirable for two reasons:

1. **Only rarely will enough information be available to prepare detailed income statements for both alternatives.**
2. **Mingling irrelevant costs with relevant costs may cause confusion and distract attention away from the information that is really critical.**

Learning Objective 2

Prepare an analysis showing whether a product line or other business segment should be dropped or retained.



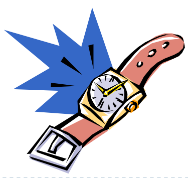
Adding/Dropping Segments

One of the most important decisions managers make is whether to add or drop a business segment. Ultimately, a decision to drop an old segment or add a new one is going to hinge primarily on the impact the decision will have on net operating income.

To assess this impact, it is necessary to carefully analyze the costs.

Adding/Dropping Segments

Due to the declining popularity of digital watches, Lovell Company's digital watch line has not reported a profit for several years. Lovell is considering discontinuing this product line.



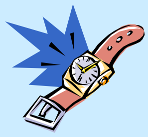
A Contribution Margin Approach

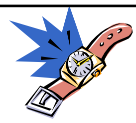
DECISION RULE

Lovell should drop the digital watch segment only if its profit would increase.

Lovell will compare the contribution margin that would be lost to the costs that would be avoided if the line was to be dropped.

Let's look at this solution.



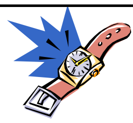


Adding/Dropping Segments

Segment Income Statement
Digital Watches

Sales		$ 500,000
Less: variable expenses		
Variable manufacturing costs	$ 120,000	
Variable shipping costs	5,000	
Commissions	75,000	200,000
Contribution margin		$ 300,000
Less: fixed expenses		
General factory overhead	$ 60,000	
Salary of line manager	90,000	
Depreciation of equipment	50,000	
Advertising - direct	100,000	
Rent - factory space	70,000	
General admin. expenses	30,000	400,000
Net operating loss		$ (100,000)

Adding/Dropping Segments



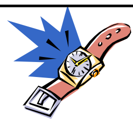
Segment Income Statement
Digital Watches

Sales		$ 500,000

An investigation has revealed that the fixed general factory overhead and fixed general administrative expenses will not be affected by dropping the digital watch line. The fixed general factory overhead and general administrative expenses assigned to this product would be reallocated to other product lines.

Advertising - direct	100,000	
Rent - factory space	70,000	
General admin. expenses	30,000	400,000
Net operating loss		$ (100,000)

Adding/Dropping Segments



Segment Income Statement
Digital Watches

Sales		$ 500,000
L		
		200,000
C		$ 300,000
Less: fixed expenses		
General factory overhead	$ 60,000	
Salary of line manag		
Depreciation of equ		
Advertising - direct		
Rent - factory space		
General admin. expenses	30,000	400,000
Net operating loss		$ (100,000)

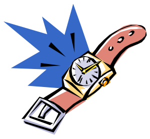
The equipment used to manufacture digital watches has no resale value or alternative use.

Should Lovell retain or drop the digital watch segment?

A Contribution Margin Approach

Contribution Margin Solution

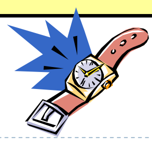
Contribution margin lost if digital watches are dropped		$ (300,000)
Less fixed costs that can be avoided		
Salary of the line manager	$ 90,000	
Advertising - direct	100,000	
Rent - factory space	70,000	260,000
Net disadvantage		$ (40,000)



Comparative Income Approach

The Lovell solution can also be obtained by preparing comparative income statements showing results with and without the digital watch segment.

Let's look at this second approach.



Comparative Income Approach
Solution

	Keep Digital Watches	Drop Digital Watches	Difference
Sales	$ 500,000	$ -	$ (500,000)
Less variable expenses:		-	
Manufacturing expenses	120,000	-	120,000
Shipping	5,000	-	5,000
Commissions	75,000	-	75,000
Total variable expenses	200,000	-	200,000
Contribution margin	300,000	-	(300,000)
Less fixed expenses:			
General factory overhead	60,000		
Salary of line manager	90,000		
Depreciation	50,000		
Advertising - direct	100,000		
Rent - factory space	70,000		
General admin. expenses	30,000		
Total fixed expenses	400,000		
Net operating loss	$ (100,000)		

If the digital watch line is dropped, the company loses $300,000 in contribution margin.

Comparative Income Approach
Solution

	Keep Digital Watches	Drop Digital Watches	Difference
Sales	$ 500,000	$ -	$ (500,000)
Less variable expenses:		-	
Manufacturing expenses	120,000	-	120,000
Shipping	5,000	-	5,000
Commissions	75,000	-	75,000
Total variable expenses	200,000	-	200,000
Contribution margin	300,000	-	(300,000)
Less fixed expenses:			
General factory overhead	60,000	60,000	-
Salary of line manager	90,000		
Depreciation			
Advertising - direct			
Rent - factory space			
General admin. expenses			
Total fixed expenses			
Net operating loss			

On the other hand, the general factory overhead would be the same under both alternatives, so it is irrelevant.

Comparative Income Approach
Solution

	Keep Digital Watches	Drop Digital Watches	Difference
Sales	$ 500,000	$ -	$ (500,000)
Less variable expenses:			
Manufacturing expens			120,000
Shipping			5,000
Commissions			75,000
Total variable expenses			200,000
Contribution margin	300,000	-	(300,000)
Less fixed expenses:			
General factory overhead	60,000	60,000	-
Salary of line manager	90,000	-	90,000
Depreciation	50,000		
Advertising - direct	100,000		
Rent - factory space	70,000		
General admin. expenses	30,000		
Total fixed expenses	400,000		
Net operating loss	$ (100,000)		

The salary of the product line manager would disappear, so it is relevant to the decision.

Comparative Income Approach
Solution

	Keep Digital Watches	Drop Digital Watches	Difference
Contribution margin	300,000	-	(300,000)
Less fixed expenses:			
General factory overhead	60,000	60,000	-
Salary of line manager	90,000		90,000
Depreciation	50,000	50,000	-
Advertising - direct	100,000		
Rent - factory space	70,000		
General admin. expenses	30,000		
Total fixed expenses	400,000		
Net operating loss	$ (100,000)		

The depreciation is a sunk cost. Also, remember that the equipment has no resale value or alternative use, so the equipment and the depreciation expense associated with it are irrelevant to the decision.

Comparative Income Approach Solution

	Keep Digital Watches	Drop Digital Watches	Difference
Sales	$ 500,000	$ -	$ (500,000)
Less variable expenses:		-	
Manufacturing expense			20,000
Shipping			5,000
Commissions			75,000
Total variable expenses			00,000
Contribution margin			00,000)
Less fixed expenses:			
General factory overhe			-
Salary of line manager			90,000
Depreciation	50,000	50,000	-
Advertising - direct	100,000	-	100,000
Rent - factory space	70,000	-	70,000
General admin. expenses	30,000	30,000	-
Total fixed expenses	400,000	140,000	260,000
Net operating loss	$ (100,000)	$ (140,000)	$ (40,000)

The complete comparative income statements reveal that Lovell would earn $40,000 of additional profit by retaining the digital watch line.

Beware of Allocated Fixed Costs

Beware of Allocated Fixed Costs

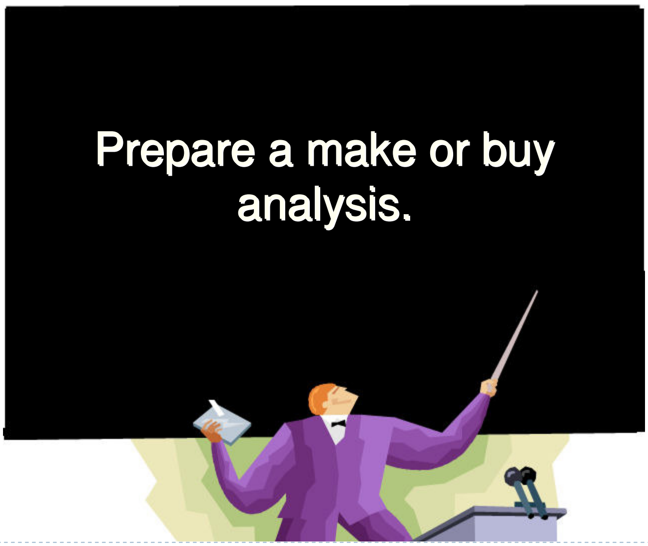
The answer lies in the way we allocate common fixed costs to our products.

Beware of Allocated Fixed Costs

Including unavoidable common fixed costs makes the product line appear to be unprofitable.

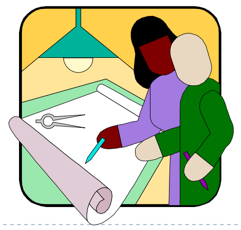
Our allocations can make a segment look less profitable than it really is.

Learning Objective 3



The Make or Buy Decision

When a company is involved in more than one activity in the entire value chain, it is vertically integrated. A decision to carry out one of the activities in the value chain internally, rather than to buy externally from a supplier is called a "make or buy" decision.



Vertical Integration- Advantages

Smoother flow of parts and materials

Better quality control

Realize profits

Vertical Integration- Disadvantage

Companies may fail to take advantage of suppliers who can create economies of scale advantage by pooling demand from numerous companies.

While the economics of scale factor can be appealing, a company must be careful to retain control over activities that are essential to maintaining its competitive position.

The Make or Buy Decision: An Example

- Essex Company manufactures part 4A that is used in one of its products.
- The unit product cost of this part is:

Direct materials	$ 9
Direct labor	5
Variable overhead	1
Depreciation of special equip.	3
Supervisor's salary	2
General factory overhead	10
Unit product cost	$ 30

The Make or Buy Decision

- The special equipment used to manufacture part 4A has no resale value.
- The total amount of general factory overhead, which is allocated on the basis of direct labor hours, would be unaffected by this decision.
- The $30 unit product cost is based on 20,000 parts produced each year.
- An outside supplier has offered to provide the 20,000 parts at a cost of $25 per part.

Should we accept the supplier's offer?

The Make or Buy Decision

	Cost Per Unit	Cost of 20,000 Units	
		Make	Buy
Outside purchase price	$ 25		$ 500,000
Direct materials (20,000 units)	$ 9	180,000	
Direct labor	5	100,000	
Variable overhead	1	20,000	
Depreciation of equip.	3	-	
Supervisor's salary	2	40,000	
General factory overhead	10	-	
Total cost	$ 30	$ 340,000	$ 500,000

The avoidable costs associated with making part 4A include direct materials, direct labor, variable overhead, and the supervisor's salary.

The Make or Buy Decision

	Cost Per Unit	Cost of 20,000 Units	
		Make	Buy
Outside purchase price	$ 25		$ 500,000
Direct materials (20,000 units)	$ 9	180,000	
Direct labor	5	100,000	
Variable overhead	1	20,000	
Depreciation of equip.	3	-	
Supervisor's salary	2	40,000	
General factory overhead	10	-	
Total cost	$ 30	$ 340,000	$ 500,000

The depreciation of the special equipment represents a sunk cost. The equipment has no resale value, thus its cost and associated depreciation are irrelevant to the decision.

The Make or Buy Decision

	Cost Per Unit	Cost of 20,000 Units Make	Cost of 20,000 Units Buy
Outside purchase price	$ 25		$ 500,000
Direct materials (20,000 units)	$ 9	180,000	
Direct labor	5	100,000	
Variable overhead	1	20,000	
Depreciation of equip.	3	-	
Supervisor's salary	2	40,000	
General factory overhead	10	-	
Total cost	$ 30	$ 340,000	$ 500,000

Not avoidable; irrelevant. If the product is dropped, it will be reallocated to other products.

The Make or Buy Decision

	Cost Per Unit	Cost of 20,000 Units Make	Cost of 20,000 Units Buy
Outside purchase price	$ 25		$ 500,000
Direct materials (20,000 units)	$ 9	180,000	
Direct labor	5	100,000	
Variable overhead	1	20,000	
Depreciation of equip.	3	-	
Supervisor's salary	2	40,000	
General factory overhead	10	-	
Total cost	$ 30	$ 340,000	$ 500,000

Should we make or buy part 4A? Given that the total avoidable costs are less than the cost of buying the part, Essex should continue to make the part.

Opportunity Cost

An opportunity cost is the benefit that is foregone as a result of pursuing some course of action.

Opportunity costs are not actual cash outlays and are not recorded in the formal accounts of an organization.

How would this concept potentially relate to the Essex Company?

Value to Business (Deprival Value)

Deprival value
= Lower of

Replacement Cost (RC)

Recoverable Value (RV)
= Higher of

Net Realizable Value (NRV)

Value in Use (VIU)
Economic Value (EV) or Present Value (PV)

Opportunity Costs: An Example

Fed Co. Ltd. is considering the publication of a limited edition of a book, bound in a special grade of leather.

- The leather was bought some year ago for $500.
- The current price for the same quantity of leather would be $1,200
- Fed Co. Ltd. Can use the leather to cover desk furnishings, in replacement for other material which would cost $400
- Fed Co. Ltd. can sell it with a net disposal proceeds of $300

How much should the leather be valued for the use of book bounding?

Opportunity Cost: Deprival Value Approach

Deprival value
= Lower of
$400

Replacement Cost
(RC)
$1,200

Recoverable Value
(RV)
= Higher of
$400

Net Realizable Value
(NRV)
$300

Value in Use (VIU)
(Economic Value (EV))
$400

Learning Objective 4

Prepare an analysis showing whether a special order should be accepted.

Key Terms and Concepts

A **special order** is a one-time order that is not considered part of the company's normal ongoing business.

When analyzing a special order, only the **incremental costs and benefits** are relevant.

Since the existing fixed manufacturing overhead costs would not be affected by the order, they are not relevant.

Special Orders

- Jet Corporation. makes a single product whose normal selling price is $20 per unit.
- A foreign distributor offers to purchase 3,000 units for $10 per unit.
- This is a one-time order that would not affect the company's regular business.
- Annual capacity is 10,000 units, but Jet Corporation is currently producing and selling only 5,000 units.

Should Jet accept the offer?

Special Orders

Jet Corporation
Contribution Income Statement

Revenue (5,000 × $20)		$ 100,000
Variable costs:		
Direct materials	$ 20,000	
Direct labor	5,000	
Manufacturing overhead	10,000	
Marketing costs	5,000	
Total variable costs		40,000
Contribution margin		60,000
Fixed costs:		
Manufacturing overhead	$ 28,000	
Marketing costs	20,000	
Total fixed costs		48,000
Net operating income		$ 12,000

$8 variable cost

Special Orders

If Jet accepts the special order, the incremental revenue will exceed the incremental costs. In other words, net operating income will increase by $6,000. This suggests that Jet should accept the order.

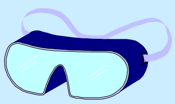
Increase in revenue (3,000 × $10)	$30,000
Increase in costs (3,000 × $8 variable cost)	24,000
Increase in net income	$ 6,000

Note: This answer assumes that the fixed costs are unavoidable and that variable marketing costs must be incurred on the special order.

Quick Check ✓

Northern Optical ordinarily sells the X-lens for $50. The variable production cost is $10, the fixed production cost is $18 per unit, and the variable selling cost is $1. A customer has requested a special order for 10,000 units of the X-lens to be imprinted with the customer's logo. This special order would not involve any selling costs, but Northern Optical would have to purchase an imprinting machine for $50,000.

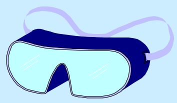
(see the next page)

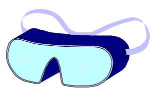


Quick Check ✓

What is the rock bottom minimum price below which Northern Optical should not go in its negotiations with the customer? In other words, below what price would Northern Optical actually be losing money on the sale? There is ample idle capacity to fulfill the order and the imprinting machine has no further use after this order.

a. $50
b. $10
c. $15
d. $29





Quick Check ✓

What is the rock bottom minimum price below which Northern Optical should not go in its negotiations with the customer? In other words, below what price would Northern Optical actually be losing money on the sale? There is ample idle capacity to fulfill the order and the imprinting machine has no further use after this order.

a. $50
b. $10
c. $15
d. $29

Variable production cost	$100,000
Additional fixed cost	+ 50,000
Total relevant cost	$150,000
Number of units	10,000
Average cost per unit=	$15

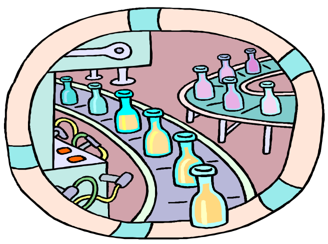
Learning Objective 5

Determine the most profitable use of a constrained resource and the value of obtaining more of the constrained resource.

Key Terms and Concepts

When a limited resource of some type restricts the company's ability to satisfy demand, the company is said to have a **constraint.**

The machine or process that is limiting overall output is called the **bottleneck** – it is the constraint.



Utilization of a Constrained Resource

- Fixed costs are usually unaffected in these situations, so the product mix that maximizes the company's total contribution margin should ordinarily be selected.
- A company should not necessarily promote those products that have the highest unit contribution margins.
- Rather, total contribution margin will be maximized by promoting those products or accepting those orders that provide the highest contribution margin in relation to the constraining resource.

Utilization of a Constrained Resource: An Example

Ensign Company produces two products and selected data are shown below:

	Product 1	Product 2
Selling price per unit	$ 60	$ 50
Less variable expenses per unit	36	35
Contribution margin per unit	$ 24	$ 15
Current demand per week (units)	2,000	2,200
Contribution margin ratio	40%	30%
Processing time required on machine A1 per unit	1.00 min.	0.50 min.

Utilization of a Constrained Resource: An Example

- Machine A1 is the constrained resource and is being used at 100% of its capacity.
- There is excess capacity on all other machines.
- Machine A1 has a capacity of 2,400 minutes per week.

Should Ensign focus its efforts on Product 1 or Product 2?

Quick Check ✓

How many units of each product can be processed through Machine A1 in one minute?

	Product 1	Product 2
a.	1 unit	0.5 unit
b.	1 unit	2.0 units
c.	2 units	1.0 unit
d.	2 units	0.5 unit

Quick Check ✓

How many units of each product can be processed through Machine A1 in one minute?

	Product 1	Product 2
a.	1 unit	0.5 unit
b.	1 unit	2.0 units
c.	2 units	1.0 unit
d.	2 units	0.5 unit

Just checking to make sure you are with us.

Quick Check ✓

What generates more profit for the company, using one minute of machine A1 to process Product 1 or using one minute of machine A1 to process Product 2?

a. Product 1

b. Product 2

c. They both would generate the same profit.

d. Cannot be determined.

Quick Check ✓

With one minute of machine A1, we could make 1 unit of Product 1, with a contribution margin of $24, or 2 units of Product 2, each with a contribution margin of $15.

$2 \times \$15 = \$30 > \$24$

b. Product 2

c. They both would generate the same profit.

d. Cannot be determined.

Utilization of a Constrained Resource

The key is the contribution margin per unit of the constrained resource.

		Product 1			Product 2	
Contribution margin per unit		$ 24			$ 15	
Time required to produce one unit	÷	1.00	min.	÷	0.50	min.
Contribution margin per minute		$ 24			$ 30	

Ensign should emphasize Product 2 because it generates a contribution margin of $30 per minute of the constrained resource relative to $24 per minute for Product 1.

Utilization of a Constrained Resource

The key is the contribution margin per unit of the constrained resource.

		Product		
		1		2
Contribution margin per unit		$ 24		$ 15
Time required to produce one unit	÷	1.00 min.	÷	0.50 min.
Contribution margin per minute		$ 24		$ 30

Ensign can maximize its contribution margin by first producing Product 2 to meet customer demand and then using any remaining capacity to produce Product 1. The calculations would be performed as follows.

Utilization of a Constrained Resource

Let's see how this plan would work.

Alloting Our Constrained Resource (Machine A1)

Weekly demand for Product 2		2,200	units
Time required per unit	×	0.50	min.
Total time required to make Product 2		1,100	min.

Utilization of a Constrained Resource

Let's see how this plan would work.

Alloting Our Constrained Resource (Machine A1)

Weekly demand for Product 2		2,200	units
Time required per unit	×	0.50	min.
Total time required to make Product 2		1,100	min.
Total time available		2,400	min.
Time used to make Product 2		1,100	min.
Time available for Product 1		1,300	min.

Utilization of a Constrained Resource

Let's see how this plan would work.

Alloting Our Constrained Resource (Machine A1)

Weekly demand for Product 2		2,200	units
Time required per unit	×	0.50	min.
Total time required to make Product 2		1,100	min.
Total time available		2,400	min.
Time used to make Product 2		1,100	min.
Time available for Product 1		1,300	min.
Time required per unit	÷	1.00	min.
Production of Product 1		1,300	units

Utilization of a Constrained Resource

According to the plan, we will produce 2,200 units of Product 2 and 1,300 of Product 1. Our contribution margin looks like this.

	Product 1	Product 2
Production and sales (units)	1,300	2,200
Contribution margin per unit	$ 24	$ 15
Total contribution margin	$ 31,200	$ 33,000

The total contribution margin for Ensign is $64,200.

Quick Check ✓

Colonial Heritage makes reproduction colonial furniture from select hardwoods.

	Chairs	*Tables*
Selling price per unit	$80	$400
Variable cost per unit	$30	$200
Board feet per unit	2	10
Monthly demand	600	100

The company's supplier of hardwood will only be able to supply 2,000 board feet this month. Is this enough hardwood to satisfy demand?

a. Yes

b. No

Quick Check ✓

Colonial Heritage makes reproduction colonial furniture from select hardwoods.

	Chairs	*Tables*
Selling price per unit	$80	$400
Variable cost per unit	$30	$200
Board feet per unit	2	10
Monthly demand	600	100

The company's supplier of hardwood will only be able to supply 2,000 board feet this month. Is this enough hardwood to satisfy demand?

a. Yes

b. No

$(2 \times 600) + (10 \times 100) = 2{,}200 > 2{,}000$

Quick Check ✓

	Chairs	*Tables*
Selling price per unit	$80	$400
Variable cost per unit	$30	$200
Board feet per unit	2	10
Monthly demand	600	100

The company's supplier of hardwood will only be able to supply 2,000 board feet this month. What plan would maximize profits?

a. 500 chairs and 100 tables

b. 600 chairs and 80 tables

c. 500 chairs and 80 tables

d. 600 chairs and 100 tables

Quick Check ✓

Selling price
Variable cost
Board feet pe
Monthly dem

	Chairs	Tables
Selling price	$ 80	$ 400
Variable cost	30	200
Contribution margin	$ 50	$ 200
Board feet	2	10
CM per board foot	$ 25	$ 20
Production of chairs	600	
Board feet required	1,200	
Board feet remaining	800	
Board feet per table	10	
Production of tables	80	

The company's s
be able to suppl
What plan woul

a. 500 chairs and

b. 600 chairs and 80 tables

c. 500 chairs and 80 tables

d. 600 chairs and 100 tables

Quick Check ✓

As before, Colonial Heritage's supplier of hardwood will only be able to supply 2,000 board feet this month. Assume the company follows the plan we have proposed. Up to how much should Colonial Heritage be willing to pay above the usual price to obtain more hardwood?

a. $40 per board foot

b. $25 per board foot

c. $20 per board foot

d. Zero

Quick Check ✓

As before, Colonial Heritage's supplier of hardwood

The additional wood would be used to make tables. In this use, each board foot of additional wood will allow the company to earn an additional $20 of contribution margin and profit.

a. $40 per board foot

b. $25 per board foot

c. $20 per board foot

d. Zero

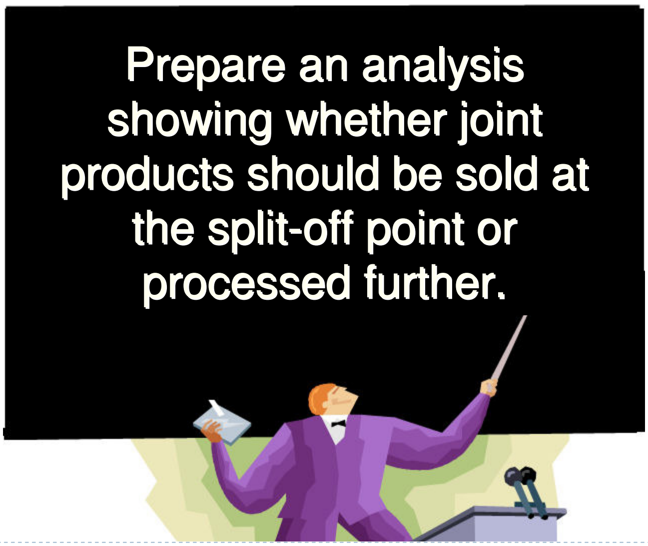
Managing Constraints

It is often possible for a manager to increase the capacity of a bottleneck, which is called relaxing (or elevating) the constraint, in numerous ways such as:

1. **Working overtime on the bottleneck.**
2. **Subcontracting some of the processing that would be done at the bottleneck.**
3. **Investing in additional machines at the bottleneck.**
4. **Shifting workers from non-bottleneck processes to the bottleneck.**
5. **Focusing business process improvement efforts on the bottleneck.**
6. **Reducing defective units processed through the bottleneck.**

These methods and ideas are all consistent with the Theory of Constraints, which was introduced in Chapter 1.

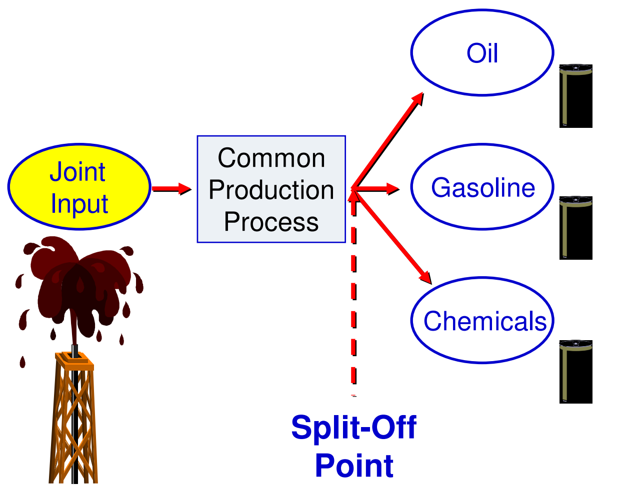
Learning Objective 6



Joint Costs

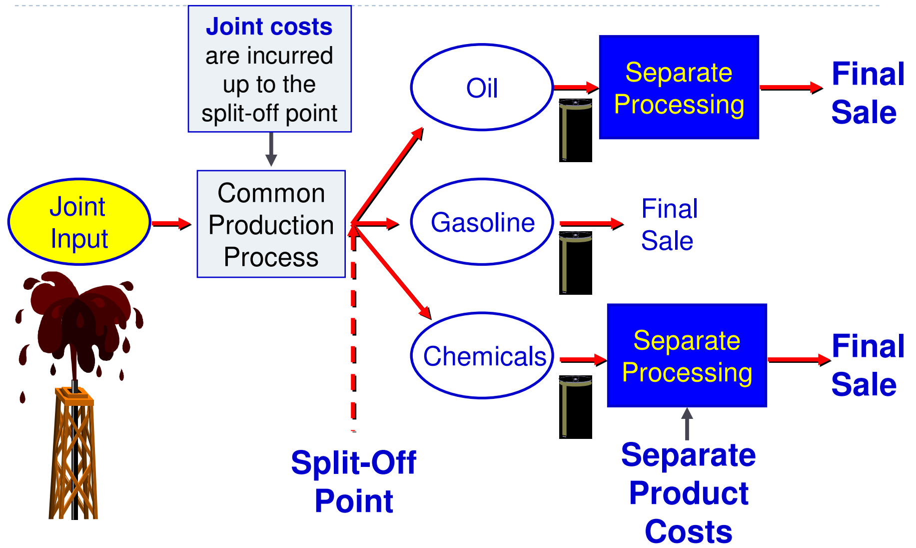
- In some industries, a number of end products are produced from a single raw material input.
- Two or more products produced from a common input are called joint products.
- The point in the manufacturing process where each joint product can be recognized as a separate product is called the split-off point.

Joint Products



For example, in the petroleum refining industry, a large number of products are extracted from crude oil, including gasoline, jet fuel, home heating oil, lubricants, asphalt, and various organic chemicals.

Joint Products



The Pitfalls of Allocation

Joint costs are traditionally allocated among different products at the split-off point. A typical approach is to allocate joint costs according to the relative sales value of the end products.

Although allocation is needed for some purposes such as balance sheet inventory valuation, allocations of this kind are very dangerous for decision making.

Sell or Process Further

Joint costs are irrelevant in decisions regarding what to do with a product from the split-off point forward. Therefore, these costs should not be allocated to end products for decision-making purposes.

With respect to sell or process further decisions, it is profitable to continue processing a joint product after the split-off point so long as the incremental revenue from such processing exceeds the incremental processing costs incurred after the split-off point.

Sell or Process Further: An Example

- Sawmill, Inc. cuts logs from which unfinished lumber and sawdust are the immediate joint products.
- Unfinished lumber is sold "as is" or processed further into finished lumber.
- Sawdust can also be sold "as is" to gardening wholesalers or processed further into "presto-logs."



Sell or Process Further

Data about Sawmill's joint products includes:

	Per Log	
	Lumber	Sawdust
Sales value at the split-off point	$ 140	$ 40
Sales value after further processing	270	50
Allocated joint product costs	176	24
Cost of further processing	50	20



Sell or Process Further

Analysis of Sell or Process Further

	Per Log	
	Lumber	Sawdust
Sales value after further processing	$ 270	$ 50
Sales value at the split-off point	140	40
Incremental revenue	130	10
Cost of further processing		
Profit (loss) from further processing		



Sell or Process Further

Analysis of Sell or Process Further

	Per Log	
	Lumber	Sawdust
Sales value after further processing	$ 270	$ 50
Sales value at the split-off point	140	40
Incremental revenue	130	10
Cost of further processing	50	20
Profit (loss) from further processing	$ 80	$ (10)



Sell or Process Further

Analysis of Sell or Process Further

	Per Log	
	Lumber	Sawdust
Sales value after further processing	$ 270	$ 50
Sales value at the split-off point	140	40
Incremental revenue	130	10
Cost of further processing	50	20
Profit (loss) from further processing	$ 80	$ (10)

The lumber should be processed further and the sawdust should be sold at the split-off point.

Activity-Based Costing and Relevant Costs

ABC can be used to help identify potentially relevant costs for decision-making purposes.

However, managers should exercise caution against reading more into this "traceability" than really exists.

People have a tendency to assume that if a cost is traceable to a segment, then the cost is automatically avoidable, which is untrue. Before making a decision, managers must decide which of the potentially relevant costs are actually avoidable.

End of Chapter 14

Short-term decisions

6

Topic list	Syllabus reference
1 Identifying relevant costs	B1 (a), (b), (c)
2 Make or buy decisions	B5 (a), (b), (c)
3 Outsourcing	B5 (a), (b), (c)
4 Further processing decisions	B5 (d)
5 Shut down decisions	B5 (d)

Introduction

The concept of **relevant costs** has already been revisited in this study text and their use in one-off contracts was examined in the last chapter.

In this chapter we look in greater depth at relevant costs and at how they should be applied in **decision-making situations**.

We look at a variety of common short-run business decisions and consider how they can be dealt with using relevant costs as appropriate.

Study guide

		Intellectual level
B1	**Relevant cost analysis**	
(a)	Explain the concept of relevant costing	2
(b)	Identify and calculate relevant costs for specific decision situations from given data	2
(c)	Explain and apply the concept of opportunity costs	2
B5	**Make-or-buy and other short-term decisions**	
(a)	Explain the issues surrounding make vs buy and outsourcing decisions	2
(b)	Calculate and compare 'make' costs with 'buy-in' costs	2
(c)	Compare in-house costs and outsource costs of completing tasks and consider other issues surrounding this decision	2
(d)	Apply relevant costing principles in situations involving shut down, one-off contracts and the further processing of joint products	2

Exam guide

The ability to recognise relevant costs and revenues is a key skill for the F5 exam and is highly examinable. Questions will be based on practical scenarios.

PER alert

One of the competencies you require to fulfil performance objective 12 of the PER is the ability to prepare management information to assist in decision making. You can apply the knowledge you obtain from this section of the text to help to demonstrate this competence.

1 Identifying relevant costs 12/11

FAST FORWARD

Relevant costs are future cash flows arising as a direct consequence of a decision.

- Relevant costs are **future costs**
- Relevant costs are **cash flows**
- Relevant costs are **incremental costs**

In this section we provide a fairly gentle introduction to the sort of thought processes that you will have to go through when you encounter a decision-making question. First some general points about machinery, labour, and particularly materials, that often catch people out.

Exam focus point

Question 1 of the December 2011 exam asked candidates to prepare a cost statement using relevant costing principles, with detailed notes to support each number included in the statement.

The examiner noted that many candidates 'just wrote down that a cost was included because it was relevant, but didn't say why'. Ensure you are able to explain **why** a cost is relevant / not relevant to a decision.

1.1 Machinery user costs

Once a machine has been bought its cost is a **sunk** cost. **Depreciation** is not a relevant cost, because it is not a cash flow. However, **using** machinery may involve some incremental costs. These costs might be referred to as **user costs** and they include hire charges and any fall in resale value of owned assets, through use.

1.1.1 Example: Machine user costs

Bronty Co is considering whether to undertake some contract work for a customer. The machinery required for the contract would be as follows.

(a) A special cutting machine will have to be hired for three months for the work (the length of the contract). Hire charges for this machine are $75 per month, with a minimum hire charge of $300.

(b) All other machinery required in the production for the contract has already been purchased by the organisation on hire purchase terms. The monthly hire purchase payments for this machinery are $500. This consists of $450 for capital repayment and $50 as an interest charge. The last hire purchase payment is to be made in two months time. The cash price of this machinery was $9,000 two years ago. It is being depreciated on a straight line basis at the rate of $200 per month. However, it still has a useful life which will enable it to be operated for another 36 months.

The machinery is highly specialised and is unlikely to be required for other, more profitable jobs over the period during which the contract work would be carried out. Although there is no immediate market for selling this machine, it is expected that a customer might be found in the future. It is further estimated that the machine would lose $200 in its eventual sale value if it is used for the contract work.

What is the relevant cost of machinery for the contract?

Solution

(a) The **cutting machine** will incur an incremental cost of $300, the minimum hire charge.

(b) The historical cost of the **other machinery** is irrelevant as a past cost; depreciation is irrelevant as a non-cash cost; and future hire purchase repayments are irrelevant because they are committed costs. The only relevant cost is the loss of resale value of the machinery, estimated at $200 through use. This 'user cost' will not arise until the machinery is eventually resold and the $200 should be discounted to allow for the time value of money. However, discounting is ignored here, and will be discussed in a later chapter.

(c) **Summary of relevant costs**

	$
Incremental hire costs	300
User cost of other machinery	200
	500

1.2 Labour

Often the labour force will be paid irrespective of the decision made and the costs are therefore **not incremental**. Take care, however, if the labour force could be put to an **alternative use**, in which case the relevant costs are the **variable costs** of the labour and associated variable overheads **plus** the **contribution forgone** from not being able to put it to its alternative use.

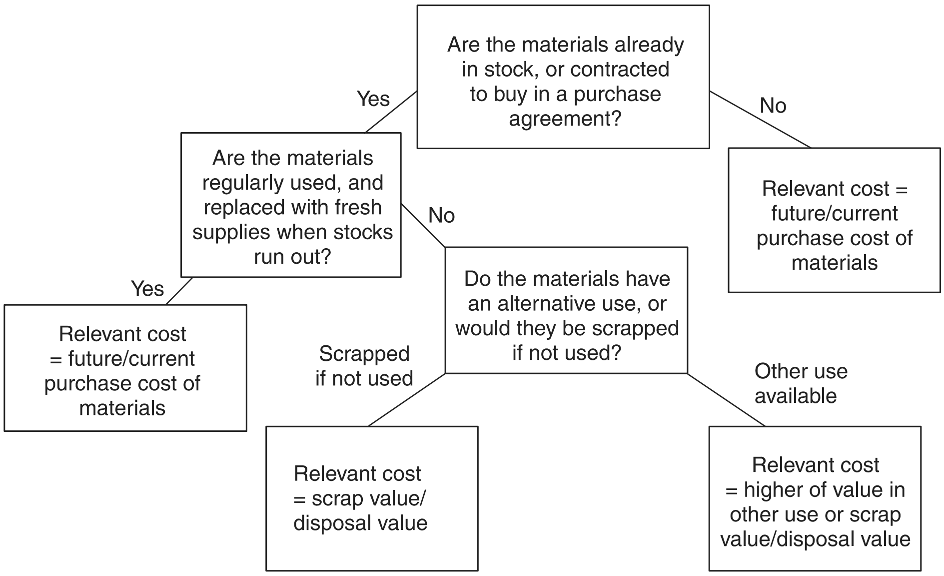
1.3 Materials

The relevant cost of raw materials is generally their current **replacement** cost, unless the materials have already been purchased and would not be replaced once used.

If materials have already been purchased but will not be replaced, then the relevant cost of using them is **either** (a) their current **resale** value **or** (b) the value they would obtain if they were put to an **alternative use**, if this is greater than their current resale value.

The **higher** of (a) or (b) is then the opportunity cost of the materials. If the materials have no resale value and no other possible use, then the relevant cost of using them for the opportunity under consideration would be nil.

The flowchart below shows how the relevant costs of materials can be identified, **provided that** the materials are **not in short supply**, and so have **no internal opportunity cost**.



Question

Relevant cost of materials

O'Reilly Co has been approached by a customer who would like a special job to be done for him, and who is willing to pay $22,000 for it. The job would require the following materials:

Material	*Total units required*	*Units already in inventory*	*Book value of units in inventory*	*Realisable value*	*Replacement cost*
			$/unit	$/unit	$/unit
A	1,000	0	–	–	6
B	1,000	600	2	2.5	5
C	1,000	700	3	2.5	4
D	200	200	4	6.0	9

(a) Material B is used regularly by O'Reilly Ltd, and if units of B are required for this job, they would need to be replaced to meet other production demand.

(b) Materials C and D are in inventory as the result of previous over-buying, and they have a restricted use. No other use could be found for material C, but the units of material D could be used in another job as substitute for 300 units of material E, which currently costs $5 per unit (of which the company has no units in inventory at the moment).

What are the relevant costs of material, in deciding whether or not to accept the contract?

Answer

(a) **Material A** is not owned and would have to be bought in full at the replacement cost of $6 per unit.

(b) **Material B** is used regularly by the company. There is existing inventory (600 units) but if these are used on the contract under review a further 600 units would be bought to replace them. Relevant costs are therefore 1,000 units at the replacement cost of $5 per unit.

(c) **Material C:** 1,000 units are needed and 700 are already in inventory. If used for the contract, a further 300 units must be bought at $4 each. The existing inventory of 700 will not be replaced. If they are used for the contract, they could not be sold at $2.50 each. The realisable value of these 700 units is an opportunity cost of sales revenue forgone.

(d) **Material D:** these are already in inventory and will not be replaced. There is an opportunity cost of using D in the contract because there are alternative opportunities either to sell the existing inventory for $6 per unit ($1,200 in total) or avoid other purchases (of material E), which would cost 300 × $5 = $1,500. Since substitution for E is more beneficial, $1,500 is the opportunity cost.

(e) **Summary of relevant costs**

	$
Material A (1,000 × $6)	6,000
Material B (1,000 × $5)	5,000
Material C (300 × $4) plus (700 × $2.50)	2,950
Material D	1,500
Total	15,450

1.4 Opportunity costs

Other potential relevant costs include **opportunity costs**.

Opportunity cost is the **benefit sacrificed** by choosing one opportunity rather than the next best alternative. You will often encounter opportunity costs when there are several possible uses for a scarce resource.

For example, if a material is in short supply, it may be transferred from the production of one product to that of another product. The opportunity cost is the **contribution lost** from ceasing production of the original product.

Key term

Opportunity cost is the value of a benefit sacrificed when one course of action is chosen, in preference to an alternative. The opportunity cost is represented by the forgone potential benefit from the best rejected course of action.

Question — Opportunity costs

An information technology consultancy firm has been asked to do an urgent job by a client, for which a price of $2,500 has been offered. The job would require the following.

(a) 30 hours' work from one member of staff, who is paid on an hourly basis, at a rate of $20 per hour, but who would normally be employed on work for clients where the charge-out rate is $45 per hour. No other member of staff is able to do the member of staff in question's work.

(b) The use of 5 hours of mainframe computer time, which the firm normally charges out to external users at a rate of $50 per hour. Mainframe computer time is currently used 24 hours a day, 7 days a week.

(c) Supplies and incidental expenses of $200.

Required

Fill in the blank in the sentence below.

The relevant cost or opportunity cost of the job is $........

Answer

The correct answer is $1,800.

The relevant cost or opportunity cost of the job would be calculated as follows.

	$
Labour (30 hours × $45)	1,350
Computer time opportunity cost (5 hours × $50)	250
Supplies and expenses	200
	1,800

2 Make or buy decisions 6/12

FAST FORWARD

In a **make or buy decision** with no limiting factors, the relevant costs are the differential costs between the two options.

A make or buy problem involves a decision by an organisation about **whether it should make a product or whether it should pay another organisation to do so**. Here are some examples of make or buy decisions.

(a) Whether a company should manufacture its own components, or else buy the components from an outside supplier
(b) Whether a construction company should do some work with its own employees, or whether it should sub-contract the work to another company
(c) Whether a service should be carried out by an internal department or whether an external organisation should be employed (discussed more fully later in this chapter)

The **'make'** option should give **management more direct control** over the work, but the **'buy'** option often has the benefit that the **external organisation** has a **specialist skill** and expertise in the work. Make or buy decisions should certainly **not be based exclusively on cost considerations.**

If an organisation has the freedom of choice about whether to make internally or buy externally and has no scarce resources that put a restriction on what it can do itself, the **relevant costs** for the decision will be the **differential costs** between the two options.

2.1 Example: Make or buy decision

Shellfish Co makes four components, W, X, Y and Z, for which costs in the forthcoming year are expected to be as follows.

	W	*X*	*Y*	*Z*
Production (units)	1,000	2,000	4,000	3,000
Unit marginal costs	$	$	$	$
Direct materials	4	5	2	4
Direct labour	8	9	4	6
Variable production overheads	2	3	1	2
	14	17	7	12

Directly attributable fixed costs per annum and committed fixed costs:

	$
Incurred as a direct consequence of making W	1,000
Incurred as a direct consequence of making X	5,000
Incurred as a direct consequence of making Y	6,000
Incurred as a direct consequence of making Z	8,000
Other fixed costs (committed)	30,000
	50,000

A sub-contractor has offered to supply units of W, X, Y and Z for $12, $21, $10 and $14 respectively. Should Shellfish make or buy the components?

Solution

(a) The **relevant costs** are the differential costs between making and buying, and they consist of **differences in unit variable costs plus differences in directly attributable fixed costs**. Sub-contracting will result in some **fixed cost savings.**

	W	X	Y	Z
	$	$	$	$
Unit variable cost of making	14	17	7	12
Unit variable cost of buying	12	21	10	14
	(2)	4	3	2
Annual requirements (units)	1,000	2,000	4,000	3,000
	$	$	$	$
Extra variable cost of buying (per annum)	(2,000)	8,000	12,000	6,000
Fixed costs saved by buying	(1,000)	(5,000)	(6,000)	(8,000)
Extra total cost of buying	(3,000)	3,000	6,000	(2,000)

(b) The company would save $3,000 pa by sub-contracting component W (where the purchase cost would be less than the marginal cost per unit to make internally) and would save $2,000 pa by sub-contracting component Z (because of the saving in fixed costs of $8,000).

(c) In this example, relevant costs are the variable costs of in-house manufacture, the variable costs of sub-contracted units, and the saving in fixed costs.

(d) **Further considerations**

   (i) If components W and Z are sub-contracted, the company will have **spare capacity**. How should that spare capacity be profitably used? Are there hidden benefits to be obtained from sub-contracting? Would the company's workforce resent the loss of work to an outside sub-contractor, and might such a decision cause an industrial dispute?

   (ii) Would the sub-contractor be **reliable** with delivery times, and would he supply components of the same **quality** as those manufactured internally?

   (iii) Does the company wish to be **flexible** and maintain better **control** over operations by making everything itself?

   (iv) Are the **estimates** of fixed cost savings reliable? In the case of Product W, buying is clearly cheaper than making in-house. In the case of product Z, the decision to buy rather than make would only be financially beneficial if it is feasible that the fixed cost savings of $8,000 will really be 'delivered' by management. All too often in practice, promised savings fail to materialise!

3 Outsourcing 12/07, 6/12

FAST FORWARD

The relevant costs/revenues in decisions relating to the **operating of internal service departments or the use of external services** are the differential costs between the two options.

3.1 The trend in outsourcing

A significant trend in the 1990s was for companies and government bodies to **concentrate on their core competences** – what they are really good at (or set up to achieve) – and turn other functions over to **specialist contractors.** A company that earns its profits from, say, manufacturing bicycles, does not also need to have expertise in, say, mass catering or office cleaning**. Facilities management** companies such as Rentokil have grown in response to this.

Key term

Outsourcing is the use of external suppliers for finished products, components or services. This is also known as **contract manufacturing** or **sub-contracting.**

Reasons for this trend include:

(a) Frequently the decision is made on the grounds that **specialist contractors** can offer **superior quality** and **efficiency**. If a contractor's main business is making a specific component it can invest in the specialist machinery and labour and knowledge skills needed to make that component. However, this component may be only one of many needed by the contractor's customer, and the complexity of components is now such that attempting to keep internal facilities up to the standard of specialists detracts from the main business of the customer.

(b) Contracting out manufacturing **frees capital** that can then be invested in core activities such as market research, product definition, product planning, marketing and sales.

(c) **Contractors** have the **capacity** and **flexibility** to start production very quickly to meet sudden **variations in demand**. In-house facilities may not be able to respond as quickly, because of the need to redirect resources from elsewhere.

3.2 Internal and external services

In administrative and support functions, too, companies are increasingly likely to use specialist companies. Decisions such as the following are now common.

(a) Whether the **design and development of a new computer system** should be entrusted to in-house data processing staff or whether an external software house should be hired to do the work.

(b) Whether **maintenance and repairs** of certain items of equipment should be dealt with by in-house engineers, or whether a maintenance contract should be made with a specialist organisation.

Even if you are not aware of specialist 'facilities management' companies such as Securicor, you will be familiar with the idea of office cleaning being done by contractors.

The costs **relevant** to such decisions are little different to those that are taken into account in a 'conventional' make or buy situation: they will be the **differential costs** between performing the service internally or using an external provider**.**

Exam focus point

The major problem in examination questions is likely to be identifying whether existing staff will be made redundant or whether they will be redeployed, and whether there are alternative uses for the other resources made available by ceasing to perform the service internally. These, it hardly needs stating, are also likely to be the major problems in practice.

3.3 Performance of outsourcers

Once a decision has been made to outsource, it is essential that the **performance** of the outsourcer is monitored and **measured**.

Measures could include cost savings, service improvement and employee satisfaction. It is important to have **realistic goals** and expectations and to have **objective ways** to measure success.

The performance of the outsourcer, whether good or bad, can interfere with the performance assessment of an **internal function**. For example:

- Maintenance of equipment could be carried out badly by an outsourcer and this may result in increased breakdowns and reduced labour efficiency of a production team
- If information arrives late or is incorrect, the wrong decision may be made

3.4 Example: Outsourcing

Stunnaz is considering a proposal to use the services of a press cuttings agency. At the moment, press cuttings are collected by a junior member of the marketing department, who is also responsible for office administration (including filing), travel bookings, a small amount of proof reading and making the tea. The total annual cost of employing this person is $15,000 pa.

There is concern that the ability of this person to produce a comprehensive file of cuttings is limited by the time available. She has calculated that she needs to spend about two hours of her seven and a half hour day simply reading the national and trade press, but usually only has about five hours a week for this job.

Press subscriptions currently cost $850 pa and are paid annually in advance.

The assistant makes use of a small micro-fiche device for storing cuttings. The cuttings are sent to a specialist firm once a month to be put onto fiche. Stunnaz pays $45 each month for this service. The micro-fiche reader is leased at a cost of $76 per calendar month. This lease has another 27 months to run.

The cuttings service bureau has proposed an annual contract at a cost of $1,250. Several existing users have confirmed their satisfaction with the service they receive.

Should Stunnaz outsource its press cuttings work?

Solution

Current annual costs amount to:

	$
Micro fiche service	$45 × 12 = 540
Subscriptions	850
	1,390

The monthly leasing charge is a **committed cost** that must be paid whatever the decision. It is not therefore a decision-relevant cost.

Engaging the services of the press cuttings agency therefore has the *potential* to save Stunnaz $140 pa. However, this is not the final word: there are other considerations.

(a) The **'in-house' option** should give management **more direct control** over the work, but the **'outsource' option** often has the benefit that the external organisation has a **specialist skill and expertise** in the work. Decisions should certainly not be based exclusively on cost considerations.

(b) Will outsourcing create **spare capacity**? How should that spare capacity be profitably used?

(c) Are there **hidden benefits** to be obtained from subcontracting?

(d) Would the company's workforce resent the loss of work to an outside subcontractor, and might such a decision cause an **industrial dispute**?

(e) Would the subcontractor be **reliable with delivery times** and **quality**?

(f) Does the company wish to be **flexible** and **maintain better control** over operations by doing everything itself?

4 Further processing decisions 12/07

FAST FORWARD

A joint product should be **processed further** past the split-off point if sales value minus post-separation (further processing) costs is greater than sales value at split-off point.

4.1 Joint products

You will have covered joint products in your earlier studies and the following will act as a brief reminder.

Knowledge brought forward from earlier studies

- **Joint products** are two or more products which are output from the same processing operation, but which are indistinguishable from each other up to their point of separation.
- Joint products have a **substantial sales value**. Often they require further processing before they are ready for sale. Joint products arise, for example, in the oil refining industry where diesel fuel, petrol, paraffin and lubricants are all produced from the same process.
- A joint product is regarded as an important saleable item, and so it should be **separately costed**. The profitability of each joint product should be assessed in the cost accounts.
- The point at which joint products become separately identifiable is known as the **split-off point** or **separation point**.

- Costs incurred prior to this point of separation are **common** or **joint costs**, and these need to be allocated (apportioned) in some manner to each of the joint products.
- Problems in **accounting** for joint products are basically of two different sorts.
  (a) How common costs should be apportioned between products, in order to put a value to closing inventory and to the cost of sale (and profit) for each product.
  (b) Whether it is more profitable to sell a joint product at one stage of processing, or to process the product further and sell it at a later stage.

Suppose a manufacturing company carries out process operations in which two or more joint products are made from a common process. If the joint products can be sold either in their existing condition at the 'split-off' point at the end of common processing or after further separate processing, **a decision should be taken about whether to sell each joint product at the split-off point or after further processing.**

Attention!

Note that **joint (pre-separation) costs** are incurred regardless of the decision and are therefore **irrelevant**.

4.2 Example: Further processing

The Poison Chemical Company produces two joint products, Alash and Pottum from the same process. Joint processing costs of $150,000 are incurred up to split-off point, when 100,000 units of Alash and 50,000 units of Pottum are produced. The selling prices at split-off point are $1.25 per unit for Alash and $2.00 per unit for Pottum.

The units of Alash could be processed further to produce 60,000 units of a new chemical, Alashplus, but at an extra fixed cost of $20,000 and variable cost of 30c per unit of input. The selling price of Alashplus would be $3.25 per unit. Should the company sell Alash or Alashplus?

Solution

The only relevant costs/incomes are those which compare selling Alash against selling Alashplus. Every other cost is irrelevant: they will be incurred regardless of what the decision is.

	Alash			*Alashplus*
Selling price per unit	$1.25			$3.25
	$		$	$
Total sales	125,000			195,000
Post-separation processing costs	–	Fixed	20,000	
	–	Variable	30,000	50,000
Sales minus post-separation (further processing) costs	125,000			145,000

It is $20,000 more profitable to convert Alash into Alashplus.

Question — Further processing decision

A company manufactures four products from an input of a raw material to Process 1. Following this process, product A is processed in Process 2, product B in Process 3, product C in Process 4 and product D in Process 5.

The normal loss in Process 1 is 10% of input, and there are no expected losses in the other processes. Scrap value in Process 1 is $0.50 per litre. The costs incurred in Process 1 are apportioned to each product according to the volume of output of each product. Production overhead is absorbed as a percentage of direct wages.

Data in respect of the month of October

	Process					
	1	*2*	*3*	*4*	*5*	*Total*
	$'000	$'000	$'000	$'000	$'000	$'000
Direct materials at $1.25 per litre	100					100
Direct wages	48	12	8	4	16	88
Production overhead						66

	Product			
	A	*B*	*C*	*D*
	litres	litres	litres	litres
Output	22,000	20,000	10,000	18,000
	$	$	$	$
Selling price	4.00	3.00	2.00	5.00
Estimated sales value at end of Process 1	2.50	2.80	1.20	3.00

Required

Suggest and evaluate an alternative production strategy which would optimise profit for the month. It should not be assumed that the output of Process 1 can be changed.

Answer

During the month, the quantity of input to Process 1 was 80,000 litres. Normal loss is 10% = 8,000 litres, and so total output should have been 72,000 litres of A, B, C and D. Instead, it was only 70,000 litres. In an 'average' month, output would have been higher, and this might have some bearing on the optimal production and selling strategy.

The **central question** is whether or not the output from Process 1 should be **processed further** in processes 2, 3, 4 and 5, or whether it should be **sold at the 'split-off' point**, at the end of Process 1. Each joint product can be looked at **individually**.

A further question is whether the **wages costs** in process 2, 3, 4 and 5 would be avoided if the joint products were sold at the end of process 1 and not processed further. It will be assumed that all the wages costs would be **avoidable**, but none of the **production overhead** costs would be. This assumption can be challenged, and in practice would have to be investigated.

	A	*B*	*C*	*D*
	$	$	$	$
Selling price, per litre	4.00	3.00	2.00	5.00
Selling price at end of process 1	2.50	2.80	1.20	3.00
Incremental selling price, per litre	1.50	0.20	0.80	2.00
Litres output	22,000	20,000	10,000	18,000
	$'000	$'000	$'000	$'000
Total incremental revenue from further processing	33	4	8	36
Avoidable costs from selling at split-off point (wages saved)	12	8	4	16
Incremental benefit/(cost) of further processing	21	(4)	4	20

This analysis would seem to indicate that **products A, C and D should be further processed** in processes 2, 4 and 5 respectively, but that **product B should be sold at the end of process 1**, without further processing in process 3. The saving would be at least $4,000 per month.

If **some production overhead** (which is 75% of direct wages) were also **avoidable**, this would mean that:

(a) Selling product B at the end of process 1 would offer further savings of up to (75% of $8,000) $6,000 in overheads, and so $10,000 in total.

(b) The incremental benefit from further processing product C might fall by up to (75% of $4,000) $3,000 to $1,000, meaning that it is only just profitable to process C beyond the split-off point.

5 Shut down decisions 12/09

FAST FORWARD

Shutdown/discontinuance problems can be simplified into short-run relevant cost decisions.

5.1 Simplifying decisions

Discontinuance or shutdown problems involve the following decisions.

(a) **Whether or not to close down** a product line, department or other activity, either because it is making losses or because it is too expensive to run

(b) If the decision is to shut down, **whether the closure should be permanent or temporary**

In practice, shutdown decisions may often involve **longer-term considerations,** and **capital expenditures and revenues.**

(a) A shutdown should result in savings in **annual operating costs** for a number of years into the future.

(b) Closure will probably release **unwanted non-current assets for sale**. Some assets might have a small scrap value, but other assets, in particular property, might have a substantial sale value.

(c) **Employees** affected by the closure must be made redundant or relocated, perhaps after retraining, or else offered early retirement. There will be lump sum payments involved which must be taken into account in the financial arithmetic. For example, suppose that the closure of a regional office would result in annual savings of $100,000, non-current assets could be sold off to earn income of $2 million, but redundancy payments would be $3 million. The shutdown decision would involve an assessment of the net capital cost of closure ($1 million) against the annual benefits ($100,000 pa).

It is possible, however, for shutdown problems to be **simplified into short-run decisions**, by making one of the following assumptions.

(a) Non-current asset sales and redundancy costs would be negligible.

(b) Income from non-current asset sales would match redundancy costs and so these capital items would be self-cancelling.

In such circumstances the financial aspect of shutdown decisions would be based on **short-run relevant costs.**

5.2 Example: Adding or deleting products (or departments)

A company manufactures three products, Pawns, Rooks and Bishops. The present net annual income from these is as follows.

	Pawns	*Rooks*	*Bishops*	*Total*
	$	$	$	$
Sales	50,000	40,000	60,000	150,000
Variable costs	30,000	25,000	35,000	90,000
Contribution	20,000	15,000	25,000	60,000
Fixed costs	17,000	18,000	20,000	55,000
Profit/loss	3,000	(3,000)	5,000	5,000

The company is concerned about its poor profit performance, and is considering whether or not to cease selling Rooks. It is felt that selling prices cannot be raised or lowered without adversely affecting net income. $5,000 of the fixed costs of Rooks are direct fixed costs which would be saved if production ceased (ie there are some attributable fixed costs). All other fixed costs, it is considered, would remain the same.

By **stopping production of Rooks**, the **consequences** would be a $10,000 fall in profits.

	$
Loss of contribution	(15,000)
Savings in fixed costs	5,000
Incremental loss	(10,000)

Suppose, however, it were possible to use the resources realised by stopping production of Rooks and **switch to producing a new item**, Crowners, which would sell for $50,000 and incur variable costs of $30,000 and extra direct fixed costs of $6,000. A new decision is now required.

	Rooks	*Crowners*
	$	$
Sales	40,000	50,000
Less variable costs	25,000	30,000
	15,000	20,000
Less direct fixed costs	5,000	6,000
Contribution to shared fixed costs and profit	10,000	14,000

It would be **more profitable to shut down production of Rooks and switch** resources to making Crowners, in order to boost profits by $4,000 to $9,000.

5.3 Timing of shutdown

An organisation may also need to consider the most appropriate **timing** for a shutdown. Some costs may be avoidable in the long run but not in the short run. For example, office space may have been rented and three months notice is required. This cost is therefore **unavoidable** for three months. In the same way supply contracts may require notice of cancellation. A month by month analysis of when notice should be given and savings will be made will help the decision making process.

5.4 Qualitative factors

As usual the decision is not merely a matter of choosing the best financial option. **Qualitative** factors must once more be considered.

(a) What impact will a shutdown decision have on employee morale?
(b) What signal will the decision give to competitors? How will they react?
(c) How will customers react? Will they lose confidence in the company's products?
(d) How will suppliers be affected? If one supplier suffers disproportionately there may be a loss of goodwill and damage to future relations.

Question — Shutdown decisions

How would the above decision change if Pawns, Rooks and Bishops were manufactured in different departments, variable costs could be split down into the costs of direct materials, labour and overheads, and fixed costs could be analysed into the costs of administrative staff and equipment and premises costs?

Answer

The decision would not change at all – unless perhaps activity based analysis of overheads were undertaken and unexpected cost patterns were revealed. The point of this exercise is to make you realise that problems that look complicated are sometimes very simple in essence even if the volume of calculations seems daunting.

5.5 Judging relative profitability

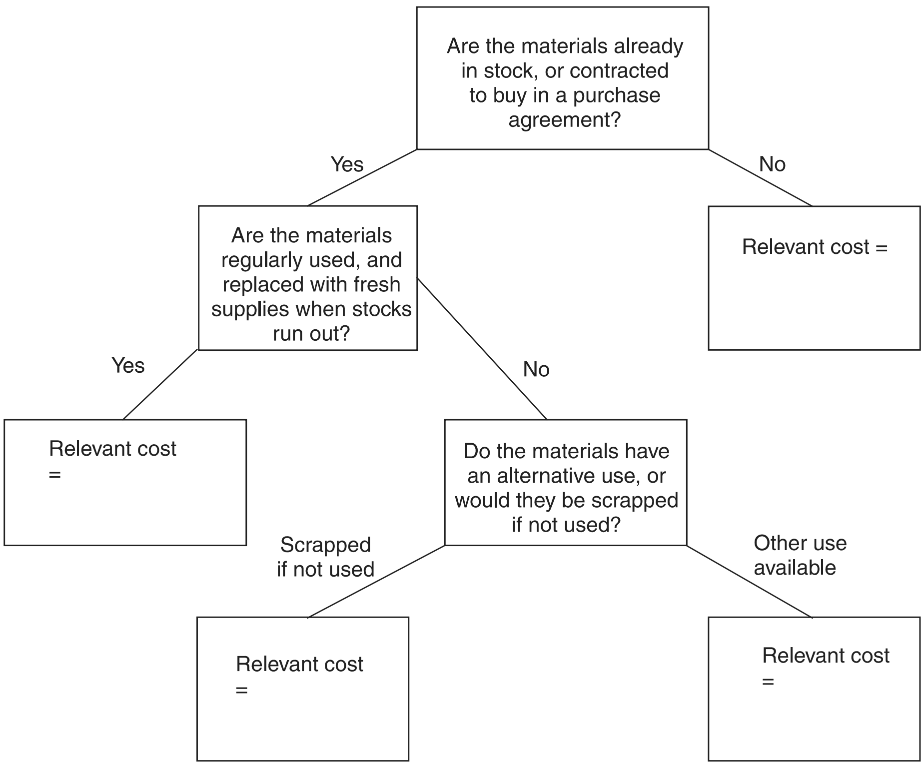
A common approach to judging the relative profitability of products is to calculate **C/S ratios**. The most profitable option is to concentrate on the product(s) with the highest C/S ratios.

Chapter Roundup

- **Relevant costs** are future cash flows arising as a direct consequence of a decision.
  - Relevant costs are **future costs**
  - Relevant costs are **incremental costs**
  - Relevant costs are **cash flows**
- In a **make or buy decision** with no limiting factors, the relevant costs are the differential costs between the two options.
- The relevant costs/revenues in decisions relating to the **operating of internal service departments or the use of external services** are the differential costs between the two options.
- A joint product should be **processed further** past the split-off point if sales value minus post-separation (further processing) costs is greater than sales value at split-off point.
- **Shutdown/discontinuance problems** can be simplified into short-run relevant cost decisions.

Quick Quiz

1 *Fill in the relevant costs in the four boxes in the diagram below.*



2 *Choose the correct word(s) from those highlighted.*

In a situation where a company must subcontract work to make up a shortfall in its own in-house capabilities, its total cost will be minimised if those units **bought out from a sub-contractor/made in-house** have the **lowest/highest** extra **variable/fixed** cost of **buying out/making in-house** per unit of **scarce resource/material.**

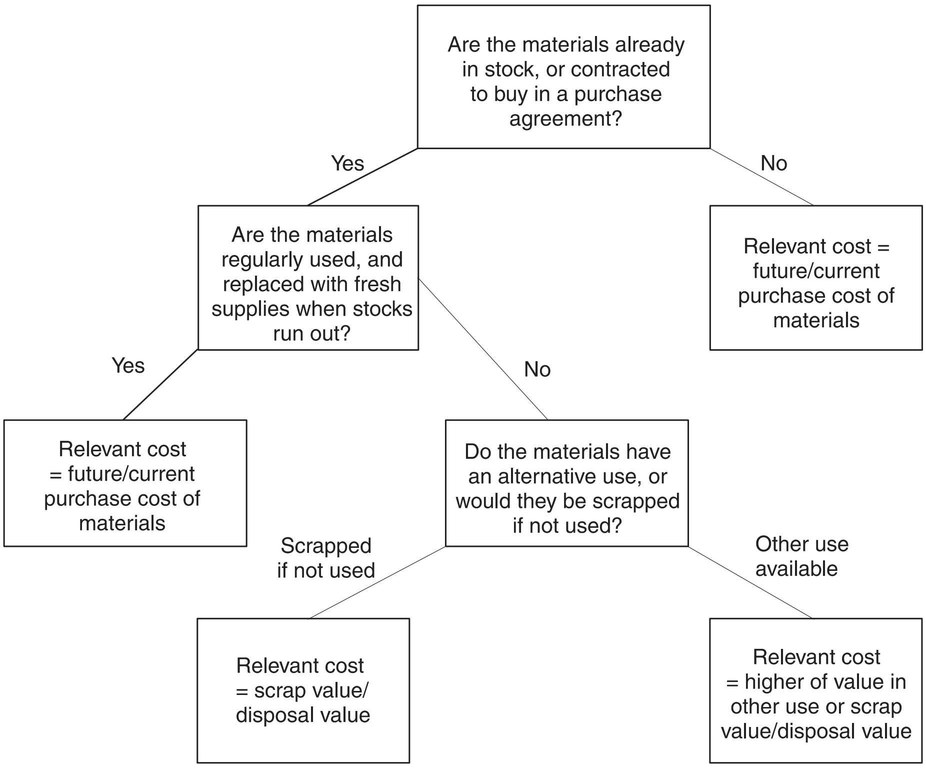
3 In a decision about whether or not to sell a joint product at the split-off point or after further processing, joint costs are relevant. **True or false?**

4 *Fill in the blanks.*

Most of the decisions considered in this chapter involve calculating …………………….. obtained from various options after identifying ……………………….. . They always involve ………………………….. issues, which depend upon the precise situation described.

Answers to Quick Quiz

1



2 bought out from a subcontractor — buying out
lowest — scarce resource
variable

3 False

4 contribution

relevant costs

qualitative

Now try the question below from the Exam Question Bank

Number	Level	Marks	Time
Q10	Examination	20	36 mins

4

Limiting factor analysis

Topic list	Syllabus reference
1 Limiting factors	B3 (a)
2 Limiting factor analysis – make or buy decisions and scarce resources	B3 (b)
3 The principles of linear programming	B3 (a)
4 The graphical method	B3 (b),(c)
5 Using simultaneous equations	B3 (c)
6 Slack and surplus	B3 (e)
7 Shadow prices	B3 (d)

Introduction

We have looked at **limiting factor analysis** in connection with throughput accounting in Chapter 2d and you will have encountered it in your earlier studies.

When there is more than one resource constraint, the technique of **linear programming** can be used. A multiple scarce resource problem can be solved using a **graphical method** and **simultaneous equations.**

We also look at the meaning and calculation of **shadow prices** and **slack.**

Study guide

		Intellectual level
B3	**Limiting factors**	
(a)	Identify limiting factors in a scarce resource situation and select an appropriate technique	2
(b)	Determine the optimal production plan where an organisation is restricted by a single limiting factor, including within the context of "make" or "buy" decisions	2
(c)	Formulate and solve a multiple scarce resource problem both graphically and using simultaneous equations as appropriate	2
(d)	Explain and calculate shadow prices (dual prices) and discuss their implications on decision-making and performance management	2
(e)	Calculate slack and explain the implications of the existence of slack for decision-making and performance management (Excluding simplex and sensitivity to changes in objective functions)	2

Exam guide

Linear programming is a popular topic in management accounting exams and is likely to be examined as a mixture of calculations and discussion. There is an article on linear programming in *Student Accountant*, March 2008. Please note that this article was not written by the current examiner.

1 Limiting factors

FAST FORWARD

All companies are limited in their capacity, either for producing goods or providing services. There is always one resource that is most restrictive (the **limiting factor**).

Key term

A **limiting factor** is any factor that is in scarce supply and that stops the organisation from expanding its activities further, that is, it limits the organisation's activities.

Knowledge brought forward from earlier studies

- An organisation might be faced with just one limiting factor (other than maximum sales demand) but there might also be several scarce resources, with two or more of them putting an effective limit on the level of activity that can be achieved.
- Examples of limiting factors include sales demand and production constraints.
  - **Labour**. The limit may be either in terms of total quantity or of particular skills.
  - **Materials**. There may be insufficient available materials to produce enough units to satisfy sales demand.
  - **Manufacturing capacity**. There may not be sufficient machine capacity for the production required to meet sales demand.
- It is assumed in limiting factor analysis that management would make a product mix decision or service mix decision based on the option that would maximise profit and that profit is maximised when contribution is maximised (given no change in fixed cost expenditure incurred). **In other words, marginal costing ideas are applied.**

- Contribution will be maximised by earning the biggest possible contribution per unit of limiting factor. For example if grade A labour is the limiting factor, contribution will be maximised by earning the biggest contribution per hour of grade A labour worked.
  - The limiting factor decision therefore involves the determination of the contribution earned per unit of limiting factor by each different product.
  - If the sales demand is limited, the profit-maximising decision will be to produce the top-ranked product(s) up to the sales demand limit.
- In limiting factor decisions, we generally assume that fixed costs are the same whatever product or service mix is selected, so that the only relevant costs are variable costs.
- When there is just one limiting factor, the technique for establishing the contribution-maximising product mix or service mix is to rank the products or services in order of contribution-earning ability per unit of limiting factor.

Exam focus point

If resources are limiting factors**, contribution** will be **maximised** by earning the biggest possible contribution per unit of limiting factor.

Where there is just one limiting factor, the technique for establishing the contribution-maximising product or service mix is to rank the products or services in order of contribution-earning ability per unit of limiting factor.

1.1 Example: Limiting factor decision

Sausage makes two products, the Mash and the Sauce. Unit variable costs are as follows.

	Mash	*Sauce*
	$	$
Direct materials	1	3
Direct labour ($3 per hour)	6	3
Variable overhead	1	1
	8	7

The sales price per unit is $14 per Mash and $11 per Sauce. During July the available direct labour is limited to 8,000 hours. Sales demand in July is expected to be as follows.

Mash	3,000 units
Sauce	5,000 units

Required

Determine the production budget that will maximise profit, assuming that fixed costs per month are $20,000 and that there is no opening inventory of finished goods or work in progress.

Solution

Step 1 Confirm that the limiting factor is something other than sales demand.

	Mash	*Sauces*	*Total*
Labour hours per unit	2 hrs	1 hr	
Sales demand	3,000 units	5,000 units	
Labour hours needed	6,000 hrs	5,000 hrs	11,000 hrs
Labour hours available			8,000 hrs
Shortfall			3,000 hrs

Labour is the limiting factor on production.

Step 2 Identify the contribution earned by each product per unit of scarce resource, that is, per labour hour worked.

	Mash	*Sauce*
	$	$
Sales price	14	11
Variable cost	8	7
Unit contribution	6	4
Labour hours per unit	2 hrs	1 hr
Contribution per labour hour (= per unit of limiting factor)	$3	$4

Although Mashes have a higher unit contribution than Sauces, two Sauces can be made in the time it takes to make one Mash. Because labour is in short supply it is more profitable to make Sauces than Mashes.

Step 3 Determine the budgeted production and sales. Sufficient Sauces will be made to meet the full sales demand, and the remaining labour hours available will then be used to make Mashes.

(a)

Product	*Demand*	*Hours required*	*Hours available*	*Priority for manufacture*
Sauces	5,000	5,000	5,000	1st
Mashes	3,000	6,000	3,000 (bal)	2nd
		11,000	8,000	

(b)

Product	*Units*	*Hours needed*	*Contribution per unit*	*Total*
			$	$
Sauces	5,000	5,000	4	20,000
Mashes (balance)	1,500	3,000	6	9,000
		8,000		29,000
Less fixed costs				20,000
Profit				9,000

Conclusion

(a) Unit contribution is *not* the correct way to decide priorities.

(b) Labour hours are the scarce resource, therefore **contribution per labour hour** is the correct way to decide priorities.

(c) The Sauce earns $4 contribution per labour hour, and the Mash earns $3 contribution per labour hour. Sauces therefore make more profitable use of the scarce resource, and should be manufactured first.

1.2 Two potentially limiting factors

You may be asked to deal with situations where two limiting factors are potentially limiting (and there are also product/service demand limitations). The approach in these situations is to find out which factor (if any) prevents the business from fulfilling maximum demand.

Exam focus point

Where there is a **maximum potential sales demand** for an organisation's products or services, they should still be ranked in order of contribution-earning ability per unit of the limiting factor. The contribution-maximising decision, however, will be to produce the top-ranked products (or to provide the top-ranked services) up to the sales demand limit.

1.3 Example: Two potentially limiting factors

Lucky manufactures and sells three products, X, Y and Z, for which budgeted sales demand, unit selling prices and unit variable costs are as follows.

		X		*Y*		*Z*	
Budgeted sales demand		550 units		500 units		400 units	
		$	$	$	$	$	$
Unit sales price			16		18		14
Variable costs:	materials	8		6		2	
	labour	4		6		9	
			12		12		11
Unit contribution			4		6		3

The organisation has existing inventory of 250 units of X and 200 units of Z, which it is quite willing to use up to meet sales demand. All three products use the same direct materials and the same type of direct labour. In the next year, the available supply of materials will be restricted to $4,800 (at cost) and the available supply of labour to $6,600 (at cost).

Required

Determine what product mix and sales mix would maximise the organisation's profits in the next year.

Solution

There **appear to be two scarce resources**, direct materials and direct labour. This is not certain, however, and because there is a limited sales demand as well, either of the following might apply.

- There is **no limiting factor at all**, except sales demand.
- There is **only one scarce resource** that prevents the full potential sales demand being achieved.

Step 1 **Establish which of the resources, if any, is scarce.**

	X	*Y*	*Z*
	Units	Units	Units
Budgeted sales	550	500	400
Inventory in hand	250	0	200
Minimum production to meet demand	300	500	200

	Minimum production to meet sales demand	*Required materials at cost*	*Required labour at cost*
	Units	$	$
X	300	2,400	1,200
Y	500	3,000	3,000
Z	200	400	1,800
Total required		5,800	6,000
Total available		4,800	6,600
(Shortfall)/Surplus		(1,000)	600

Materials are a limiting factor, but labour is not.

Step 2 **Rank** X, Y and Z in order of contribution earned per $1 of direct materials consumed.

	X	*Y*	*Z*
	$	$	$
Unit contribution	4	6	3
Cost of materials	8	6	2
Contribution per $1 materials	$0.50	$1.00	$1.50
Ranking	3rd	2nd	1st

Step 3 **Determine a production plan.** Z should be manufactured up to the limit where units produced plus units held in inventory will meet sales demand, then Y second and X third, until all the available materials are used up.

Ranking	*Product*	*Sales demand less units held* Units	*Production quantity* Units		*Materials cost* $
1st	Z	200	200	(× $2)	400
2nd	Y	500	500	(× $6)	3,000
3rd	X	300	175	(× $8)	*1,400
		Total available			4,800

* Balancing amount using up total available.

Step 4 **Draw up a budget.** The profit-maximising budget is as follows.

Product	*Units*	*Material cost / unit* $	*Total material cost* $	*Contribution per $1 of material* $	*Total contribution* $
Z op. inventory	200				
Z production	200				
	400	$2	800	$1.50	1,200
Y op. inventory	-				
Y production	500				
	500	$6	3,000	$1.00	3,000
X op. inventory	250				
X production	175				
	425	$8	3,400	$0.50	1,700
				Total contribution	5,900

2 Limiting factor analysis – make or buy decisions and scarce resources 6/11

FAST FORWARD

In a situation where a company must **sub-contract work to make up a shortfall in its own in-house capabilities**, its total costs will be minimised if those units bought have the lowest extra variable cost of buying per unit of scarce resource saved by buying.

2.1 Combining internal and external production

An organisation might **want to do more things than it has the resources for**, and so its alternatives would be as follows.

(a) Make the best use of the available resources and ignore the opportunities to buy help from outside
(b) Combine internal resources with buying externally so as to do more and increase profitability

Buying help from outside is justifiable if it **adds to profits**. A further decision is then required on how to split the work between **internal** and **external** effort. What parts of the work should be given to suppliers or sub-contractors so as to maximise profitability?

In a situation where a company must **sub-contract work to make up a shortfall in its own in-house capabilities**, its total costs will be minimised if those units bought have the lowest extra variable cost of buying per unit of scarce resource saved by buying.

2.1.1 Example: Make or buy decisions with scarce resources

MM manufactures three components, S, A and T using the same machines for each. The budget for the next year calls for the production and assembly of 4,000 of each component. The variable production cost per unit of the final product is as follows.

	Machine hours	*Variable cost*
		$
1 unit of S	3	20
1 unit of A	2	36
1 unit of T	4	24
Assembly		20
		100

Only 24,000 hours of machine time will be available during the year, and a sub-contractor has quoted the following unit prices for supplying components: S $29; A $40; T $34.

Required

Advise MM.

Solution

The organisation's budget calls for 36,000 hours of machine time, if all the components are to be produced in-house. Only 24,000 hours are available, and so there is a shortfall of 12,000 hours of machine time, which is therefore a limiting factor. The shortage can be overcome by subcontracting the equivalent of 12,000 machine hours' output to the subcontractor.

The assembly costs are not relevant costs because they are unaffected by the decision.

The decision rule is to **minimise the extra variable costs of sub-contracting per unit of scarce resource saved** (that is, per machine hour saved).

	S	*A*	*T*
	$	$	$
Variable cost of making	20	36	24
Variable cost of buying	29	40	34
Extra variable cost of buying	9	4	10
Machine hours saved by buying	3 hrs	2 hrs	4 hrs
Extra variable cost of buying per hour saved	$3	$2	$2.50

This analysis shows that it is **cheaper to buy A than to buy T** and it is **most expensive to buy S**. The **priority for making** the components in-house will be in the **reverse order**: S, then T, then A. There are enough machine hours to make all 4,000 units of S (12,000 hours) and to produce 3,000 units of T (another 12,000 hours). 12,000 hours' production of T and A must be sub-contracted.

The cost-minimising and so profit-maximising make and buy schedule is as follows.

Component		*Machine hours used/saved*	*Number of units*	*Unit variable cost*	*Total variable cost*
				$	$
Make:	S	12,000	4,000	20	80,000
	T	12,000	3,000	24	72,000
		24,000			152,000
Buy:	T	4,000	1,000	34	34,000
	A	8,000	4,000	40	160,000
		12,000			346,000
Total variable cost of components, excluding assembly costs					

Question — Make or buy and limiting factors

TW manufactures two products, the D and the E, using the same material for each. Annual demand for the D is 9,000 units, while demand for the E is 12,000 units. The variable production cost per unit of the D is $10, that of the E $15. The D requires 3.5 kgs of raw material per unit, the E requires 8 kgs of raw material per unit. Supply of raw material will be limited to 87,500 kgs during the year.

A sub contractor has quoted prices of $17 per unit for the D and $25 per unit for the E to supply the product. How many of each product should TW manufacture in order to maximise profits?

Required

Fill in the blanks in the sentence below.

TW should manufacture .... units of D and .... units of E to maximise profits.

Answer

The correct answer is: TW should manufacture 9,000 units of D and 7,000 units of E.

	D	*E*
	$ per unit	$ per unit
Variable cost of making	10	15
Variable cost of buying	17	25
Extra variable cost of buying	7	10
Raw material saved by buying	3.5 kgs	8 kgs
Extra variable cost of buying per kg saved	$2	$1.25
Priority for internal manufacture	1	2

Production plan			*Material used*
			kgs
∴ Make	D	(9,000 × 3.5 kgs)	31,500
	E	(7,000 × 8 kgs)	56,000
			87,500

The remaining 5,000 units of E should be purchased from the contractor.

3 The principles of linear programming 6/08

FAST FORWARD

Linear programming is a technique for solving problems of profit maximisation or cost minimisation and resource allocation. 'Programming' has nothing to do with computers: the word is simply used to denote a series of events. If a scenario contains **two or more limiting factors**, linear programming must be applied.

A typical business problem is to decide how a company should **divide up its production among the various types of product** it manufactures in order to obtain the **maximum possible profit**. A business cannot simply aim to produce as much as possible because there will be **limitations** or **constraints** within which the production must operate. Such constraints could be one or more of the following.

- Limited quantities of raw materials available
- A fixed number of labour hours per week for each type of worker
- Limited machine hours

Moreover, since the profits generated by different products vary, it may be better not to produce any of a less profitable line, but to concentrate all resources on producing the more profitable ones. On the other hand limitations in market demand could mean that some of the products produced may not be sold.

4 The graphical method 6/08, 6/10

FAST FORWARD

Linear programming, at least at this fairly simple level, is a technique that can be carried out in a fairly 'handle turning' manner once you have got the basic ideas sorted out. The steps involved are as follows.

1 Define variables
2 Establish constraints
3 Construct objective function
4 Graph constraints
5 Establish feasible region
6 Determine optimal solution

4.1 Example: WX Co

The following example will be used throughout the chapter to illustrate the graphical method of linear programming.

WX Co manufactures two products, A and B. Both products pass through two production departments, mixing and shaping. The organisation's objective is to maximise contribution to fixed costs.

Product A is sold for $1.50 whereas product B is priced at $2.00. There is unlimited demand for product A but demand for B is limited to 13,000 units per annum. The machine hours available in each department are restricted to 2,400 per annum. Other relevant data are as follows.

Machine hours required	*Mixing*	*Shaping*
	Hrs	Hrs
Product A	0.06	0.04
Product B	0.08	0.12

Variable cost per unit	$
Product A	1.30
Product B	1.70

Question — Constraints

What are the constraints in the situation facing WX Co?

Answer

These are the constraints that will prevent WX from producing and selling as much of each product as it chooses.

The constraints are machine hours in each department and sales demand for product B. There is no restriction on the availability of labour hours. Selling price cannot be a constraint.

4.2 Formulating the problem using the graphical method: steps 1-3

Let's formulate WX's problem.

Step 1 **Define variables**

What are the **quantities that WX can vary**? Obviously not the number of machine hours or the demand for product B. The only things which it can vary are the **number of units of each type of product produced**. It is those numbers which the company has to determine in such a way as to obtain the maximum possible profit. Our variables (which are usually products being produced) will therefore be as follows.

Let x = number of units of product A produced.

Let y = number of units of product B produced.

Step 2 **Establish constraints**

The **value of the objective function** (the maximum contribution achievable from producing products A and B) is **limited by the constraints** facing WX, however. To incorporate this into the problem we need to **translate the constraints into inequalities involving the variables** defined in Step 1. An inequality is an equation taking the form 'greater than or equal to' or 'less than or equal to'.

(a) Consider the **mixing department machine hours** constraint.

(i) **Each unit of product A** requires 0.06 hours of machine time. Producing five units therefore requires 5×0.06 hours of machine time and, more generally, **producing x units will require 0.06x hours**.

(ii) Likewise producing **y units of product B will require 0.08y hours**.

(iii) The total machine hours needed in the mixing department to make x units of product A and y units of product B is $0.06x + 0.08y$.

(iv) We know that this **cannot be greater than 2,400 hours** and so we arrive at the following inequality.

$\mathbf{0.06x + 0.08y \leq 2{,}400}$

Question — Inequalities

How can the constraint facing the **shaping department** be written as an inequality?

Answer

The constraint facing the shaping department can be written as follows:

$\mathbf{0.04x + 0.12y \leq 2{,}400}$

The constraint has to be a 'less than or equal to' inequality, because the amount of resource used ($0.04x + 0.12y$) has to be 'less than or equal to' the amount available of 2,400 hours.

(b) The final inequality is easier to obtain. The **number of units of product B produced and sold is y** but this has to be **less than or equal to 13,000**. Our inequality is therefore as follows.

$\mathbf{y \leq 13{,}000}$.

(c) We also need to add **non-negativity constraints** (**$x \geq 0$, $y \geq 0$**) since negative numbers of products cannot be produced. (Linear programming is simply a mathematical tool and so there is nothing in this method which guarantees that the answer will 'make sense'. An unprofitable product may produce an answer which is negative. This is mathematically correct but nonsense in operational terms. Always remember to include the non-negativity constraints. The examiner will not appreciate 'impossible' solutions.)

Step 3 **Construct objective function**

We now need to introduce the question of contribution or profit. We know that the **contribution on each type of product** is as follows.

			$ per unit
Product A	$(1.50 – 1.30)	=	0.20
Product B	$(2.00 – 1.70)	=	0.30

The **objective of the company is to maximise contribution** and so the **objective function to be maximised** is as follows.

Contribution (C) = 0.2x + 0.3y

The **problem** has now been reduced to the following **four inequalities** and **one equation**.

Maximise contribution (C) = 0.2x + 0.3y, subject to the following constraints:

$$0.06x + 0.08y \leq 2{,}400$$

$$0.04x + 0.12y \leq 2{,}400$$

$$y \leq 13{,}000$$

$$x,y \geq 0$$

4.3 Steps 4 and 5 – graphing the problem and establishing the feasible region

Step 4 of the linear programming model is to represent the constraints as straight lines on a graph.

A **graphical solution** is **only possible** when there are **two variables** in the problem. One variable is represented by the **x axis** of the graph and one by the **y axis**. Since non-negative values are not usually allowed, the graph shows **only zero and positive values of x and y**.

You should be familiar with graphing an equation as a straight line from your earlier studies. The key points are re-capped below.

Knowledge brought forward from earlier studies

Revision of graphing a straight line

- To begin with, we must have a linear relationship between two measurements. For example, **y = 4x + 5**
- **Note:**

  (1) To recognise a linear relationship the equation must only have 'x' not 'x' to the power of anything, such as x^2

  (2) A straight line has two characteristics:

  (i) a slope or gradient – which measures the 'steepness' of the line

  (ii) a point at which it cuts the y axis – this is called the intercept:

  y = (slope × x) + intercept. For example, **y = 4x + 5**

- Therefore, the gradient is 4 and the point at which the line cuts the y axis is 5.
- To draw a straight line graph we only need to know two points that can then be joined.
- Consider the following two equations:

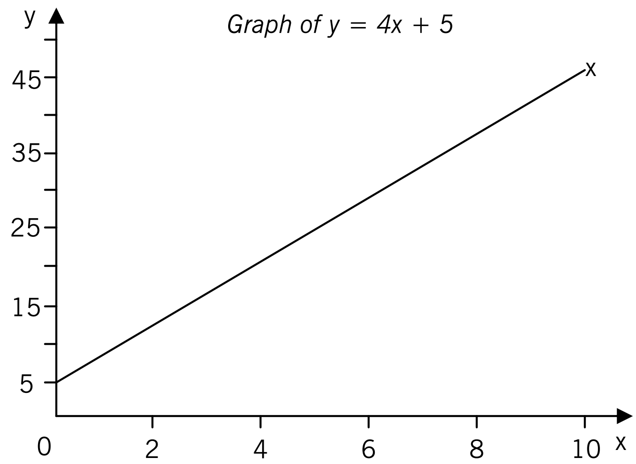
  (i) $y = 4x + 5$

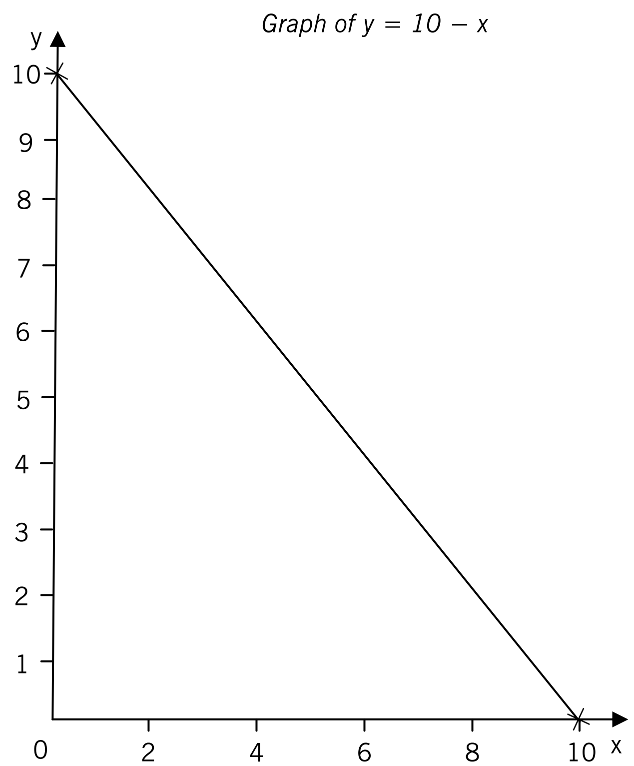
  (ii) $y = 10 - x$

- In order to draw the graphs of these equations it is necessary to decide on two values for x and then calculate the corresponding values for y. Let us use $x = 0$ and 10.

  (i) ($x = 0$, $y = 5$) and ($x = 10$, $y = 45$)
  (ii) ($x = 0$, $y = 10$) and ($x = 10$, $y = 0$)

- So to draw equation (i), we plot the points (0, 5) and (10, 45) and join them up.
- Similarly, to draw equation (ii), we plot the points (0, 10) and (10, 0) and join them up.

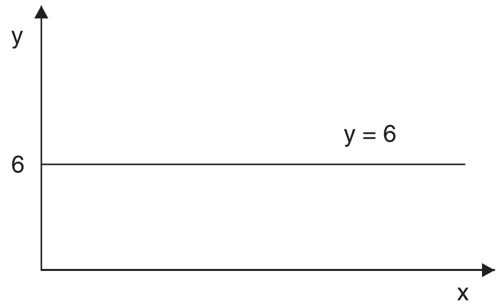




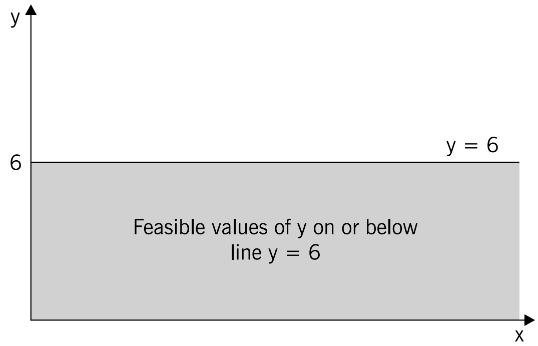
4.3.1 Graphing equations and constraints

We will return to the scenario facing WX Co at the send of section 4.3.2. For now, let us focus on the main principles involved in graphing equations and constraints.

A **linear equation with one or two variables** is shown as a **straight line on a graph**. Thus y = 6 would be shown as follows.



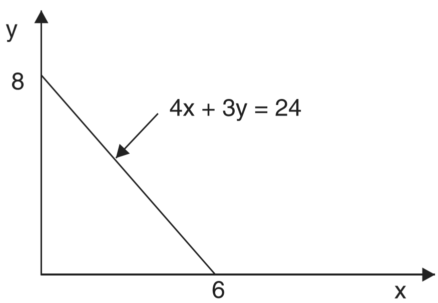
If the problem included a **constraint that y could not exceed 6**, the inequality y ≤ 6 would be represented by the **shaded area of the graph below**.



The equation 4x + 3y = 24 is also a straight line on a graph. To **draw any straight line**, we **need only to plot two points and join them up**. The easiest points to plot are the following.

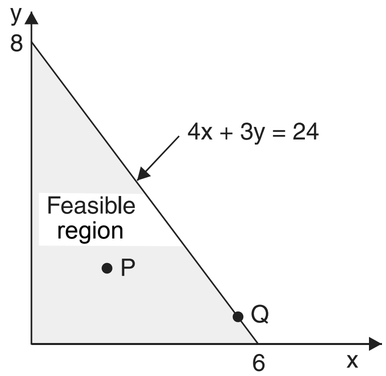
- x = 0 (in this example, if x = 0, 3y = 24, y = 8)
- y = 0 (in this example, if y = 0, 4x = 24, x = 6)

By plotting the points, (0, 8) and (6, 0) on a graph, and joining them up, we have the line for 4x + 3y = 24.



Any combination of values for x and y on the line satisfies the equation. Thus at a point where x = 3 and y = 4, 4x + 3y = 24. Similarly, at a point where x = 4.5 and y = 2, 4x + 3y = 24.

If we had a **constraint 4x + 3y ≤ 24**, **any combined value of x and y within the shaded area below (on or below the line) would satisfy the constraint.**



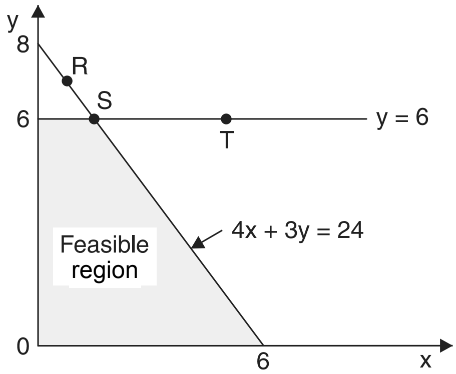
Consider point P which has coordinates of (2, 2). Here 4x + 3y = 14, which is less than 24; and at point Q where x = 5½, y = 2/3, 4x + 3y = 24. **Both P and Q lie within the feasible region** or **feasible area**.

4.3.2 Establishing the feasible region

Key term

A **feasible region** is the area contained within all of the constraint lines shown on a graphical depiction of a linear programming problem. All feasible combinations of output are contained within or located on the boundaries of the feasible region.

When there are **several constraints**, the **feasible region** of combinations of values of x and y must be an area **where all the inequalities are satisfied**. Thus, if **$y \leq 6$ and $4x + 3y \leq 24$** the **feasible region** would be the **shaded area** in the following graph.

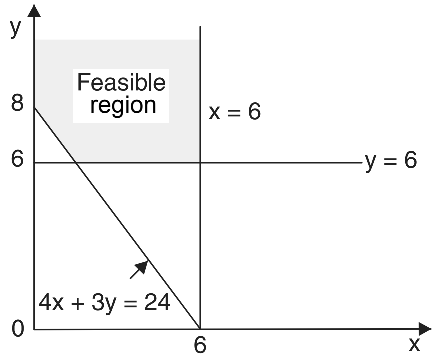


(a) Point R (x = 0.75, y = 7) is not in the feasible region because although it satisfies the inequality $4x + 3y \leq 24$, it does not satisfy $y \leq 6$.

(b) Point T (x = 5, y = 6) is not in the feasible region, because although it satisfies the inequality $y \leq 6$, it does not satisfy $4x + 3y \leq 24$.

(c) Point S (x = 1.5, y = 6) satisfies both inequalities and lies just on the boundary of the feasible region since y = 6 exactly, and 4x + 3y = 24. Point S is thus at the intersection of the two lines.

Similarly, if y ≥ 6 and 4x + 3y ≥ 24 but x is ≤ 6, the feasible region would be the shaded area in the graph below.

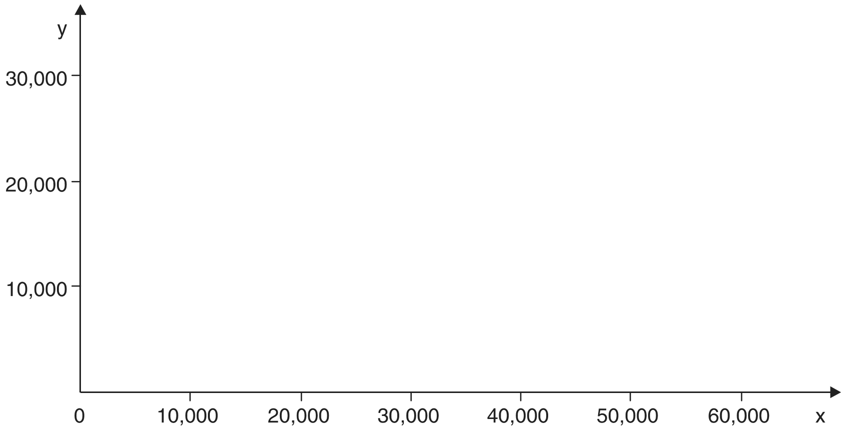


Let's return to scenario facing WX Co.

Question

Feasible region

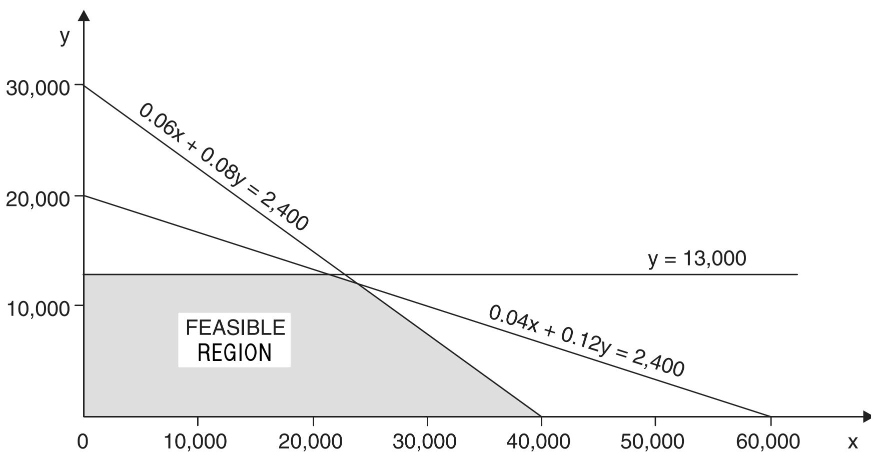
Draw the feasible region which arises from the constraints facing WX on the graph below (the constraints are listed in section 4.3.2).



Answer

If 0.06x + 0.08y = 2,400, then if x = 0, y = 30,000 and if y = 0, x = 40,000.

If 0.04x + 0.12y = 2,400, then if x = 0, y = 20,000 and if y = 0, x = 60,000.



Exam focus point

> You may be required to draw a graph as part of an exam question. Marks will be available for labels and a title so make sure you can draw a clear graph.

4.4 Step 6 – finding the optimum allocation of resources 6/08, 6/10, 12/10

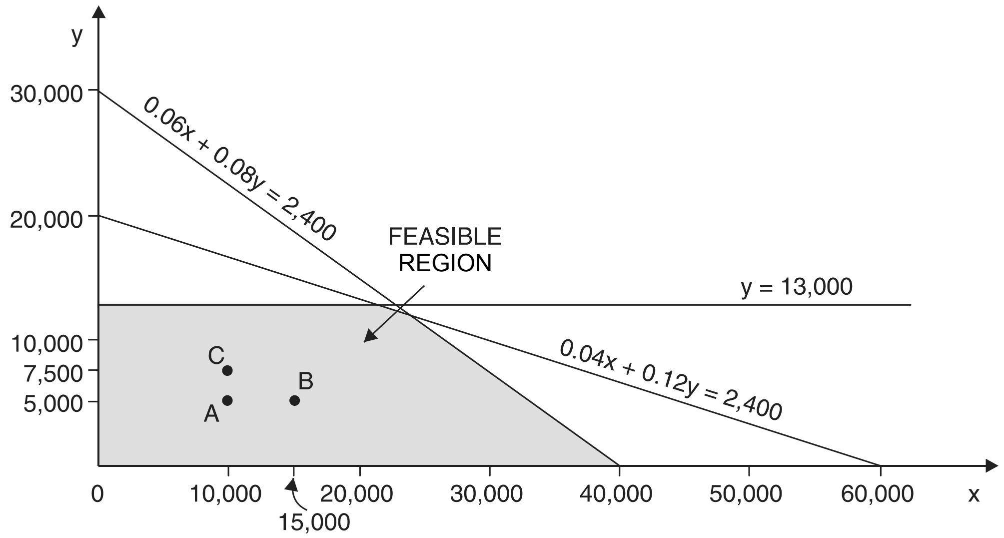
FAST FORWARD

> The **optimum solution** can be found by 'sliding the iso-contribution line out'.

Having found the feasible region (which includes all the possible solutions to the problem) we need to **find which of these possible solutions is 'best'** or **optimal** in the sense that it yields the maximum possible contribution.

Look at the feasible region of the problem faced by WX (see the answer to the question above). Even in such a simple problem as this, there are a **great many possible solution points within the feasible region.** Even to write them all down would be a time-consuming process and also an unnecessary one, as we shall see.

Here is the graph of WX's problem.



(a) Consider point A at which 10,000 units of product A and 5,000 units of product B are being manufactured. This will yield a contribution of (10,000 × \$0.20) + (5,000 × \$0.30) = \$3,500.

(b) We would clearly get more contribution at point B, where the same number of units of product B are being produced but where the number of units of product A has increased by 5,000.

(c) We would also get more contribution at point C where the number of units of product A is the same but 2,500 more units of product B are being produced.

This argument suggests that the **'best' solution** is going to be at a **point on the edge of the feasible region** rather than in the middle of it.

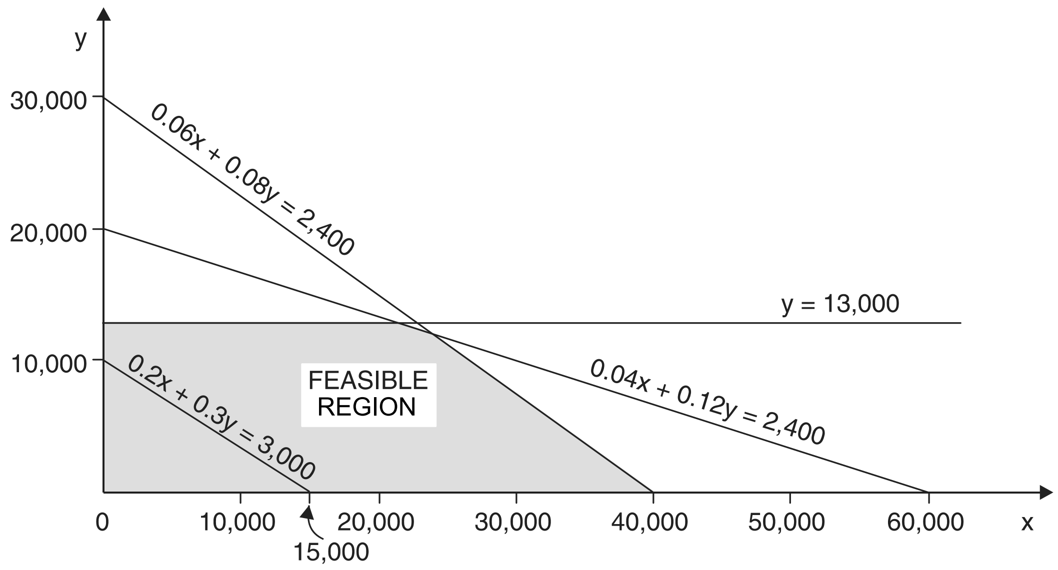
This still leaves us with quite a few points to look at but there is a way in which we can **narrow down still further the likely points at which the best solution will be found.** Suppose that WX wishes to earn contribution of \$3,000. The company could sell the following combinations of the two products.

(a) 15,000 units of A, no B.

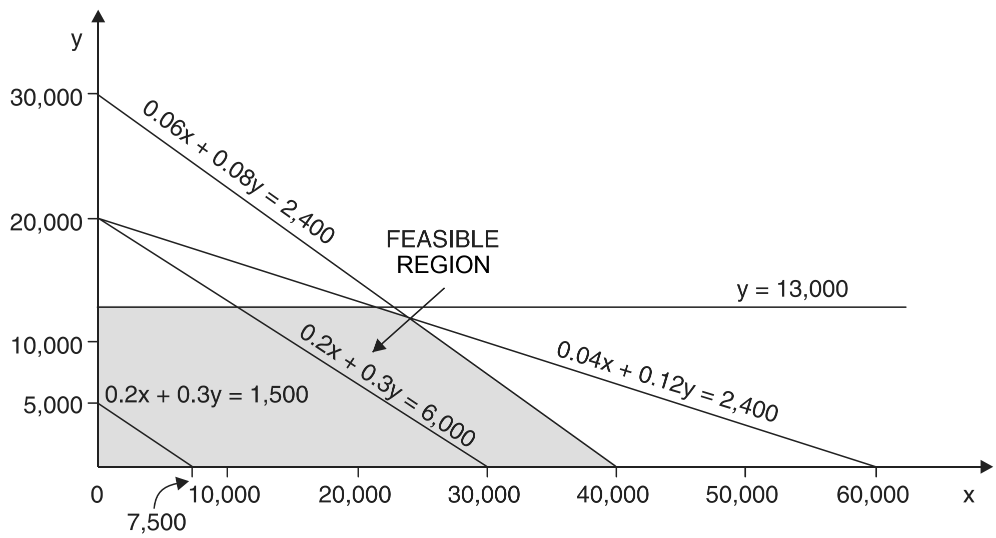
(b) No A, 10,000 units of B.

(c) A suitable mix of the two, such as 7,500 A and 5,000 B.

The **possible combinations required to earn contribution of \$3,000** could be **shown by the straight line 0.2x + 0.3y = 3,000.**



Likewise for profits of \$6,000 and \$1,500, lines of 0.2x + 0.3y = 6,000 and 0.2x + 0.3y = 1,500 could be drawn **showing the combination of the two products** which would **achieve contribution of \$6,000 or \$1,500.**



The **contribution lines are all parallel**. (They are called **iso-contribution lines**, 'iso' meaning equal.) A similar line drawn for any other total contribution would also be parallel to the three lines shown here. **Bigger contribution is shown by lines further from the origin** (0.2x + 0.3y = 6,000), smaller contribution by lines closer to the origin (0.2x + 0.3y = 1,500). As WX tries to increase possible contribution, we need to 'slide' any contribution line outwards from the origin, while always keeping it parallel to the other contribution lines.

As we do this there will come a point at which, if we were to move the contribution line out any further, it would cease to lie in the feasible region. Greater contribution could not be achieved, because of the constraints. In our example concerning WX this will happen, as you should test for yourself, where the contribution line just passes through the intersection of 0.06x + 0.08y = 2,400 and 0.04x + 0.12y = 2,400 (at coordinates (24,000, 12,000)).

The point (24,000, 12,000) will therefore give us the optimal allocation of resources (**to produce 24,000 units of A and 12,000 units of B**).

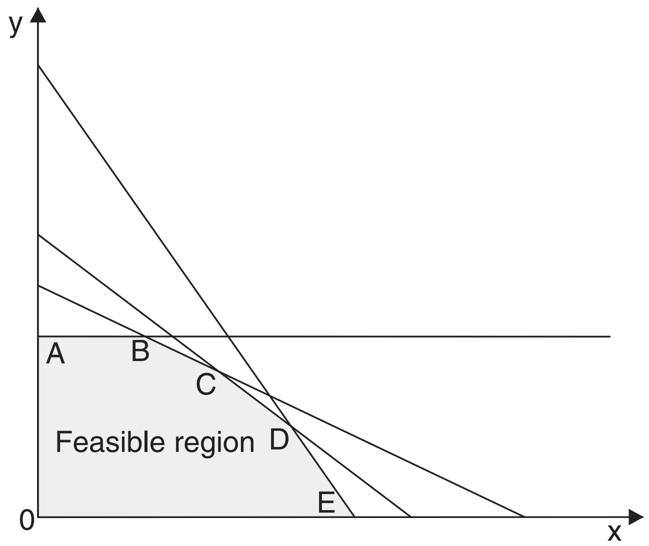
5 Using simultaneous equations 6/08

FAST FORWARD

The optimal solution can also be found using **simultaneous equations**.

You might think that a lot of time could be saved if we started by solving the simultaneous equations in a linear programming problem and did not bother to draw the graph.

Certainly, this procedure may give the right answer, but in general, it is *not* recommended until you have shown graphically which constraints are effective in determining the optimal solution. (In particular, if a question requires 'the graphical method', you *must* draw a graph). To illustrate this point, consider the following graph.



No figures have been given on the graph but the feasible region is OABCDE. When solving this problem, we would know that the optimum solution would be at one of the corners of the feasible area. We need to work out the profit at each of the corners of the feasible area and pick the one where the profit is greatest.

Once the optimum point has been determined graphically, simultaneous equations can be applied to find the exact values of x and y at this point.

Instead of a 'sliding the contribution line out' approach, **simultaneous equations** can be used to determine the optimal allocation of resources, as shown in the following example.

5.1 Example: Using simultaneous equations

The six-step process for WX Co is summarised below. Let's focus on **step 6** to illustrate how simultaneous equations can be used to establish the contribution-maximising product mix.

Step 1 **Define variables**

x = number of units of product A produced

y = number of units of product B produced

Step 2 **Establish constraints**

The constraints are as follows.

$0.06x + 0.08y \leq 2{,}400$	(mixing department)
$0.04x + 0.12y \leq 2{,}400$	(shaping department)
$y \leq 13{,}000$	(demand for product B)
$x, y \geq 0$	(non-negativity)

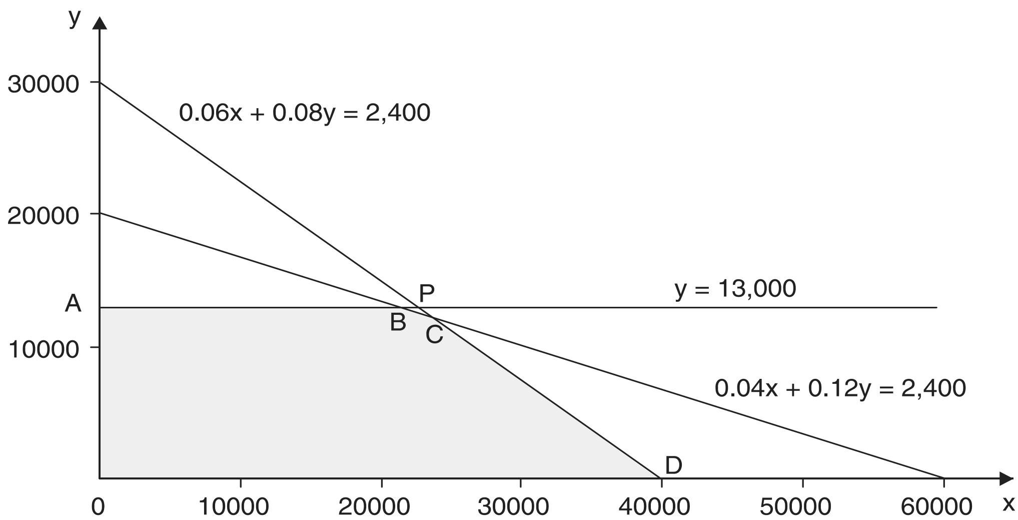
Step 3 **Construct objective function**

Product A yields a contribution of \$0.20 per unit \$(1.50 – 1.30).

Product B yields a contribution of \$0.30 per unit \$(2.00 – 1.70).

Therefore the objective is to maximise contribution (C) = 0.2x + 0.3y subject to the constraints.

Step 4 **Graph problem**



Step 5 **Establish feasible region**

The combinations of x and y that satisfy all three constraints are represented by the area OABCD.

Step 6 **Determine optimal solution**

Which combination will maximise contribution? Obviously, the more units of x and y, the bigger the contribution will be, and the optimal solution will be at point B, C or D. It will not be at A, since at A, y = 13,000 and x = 0, whereas at B, y = 13,000 (the same) and x is greater than zero.

Using simultaneous equations to calculate the value of x and y at each of points B, C and D, and then working out total contribution at each point from this, we can establish the contribution-maximising product mix.

Point B

y	=	13,000 (1)
0.04x + 0.12y	=	2,400 (2)

Substitute (1) into (2) :

0.04x + (0.12 × 13,000)	=	2,400
0.04x + 1,560	=	2,400
0.04x	=	2,400 – 1,560
0.04x	=	840
x	=	840/0.04
x	=	21,000

Total contribution = (21,000 × \$0.20) + (13,000 × \$0.30) = \$8,100.

Point C

0.06x + 0.08y	=	2,400 (1)
0.04x + 0.12y	=	2,400 (2)
0.12x + 0.16y	=	4,800 (3) ((1) × 2)
0.12x + 0.36y	=	7,200 (4) ((2) × 3)
0.2y	=	2,400 ((4) – (3))
y	=	12,000 (6)
0.06x + 960	=	(substitute y in (1))
x	=	24,000

Total contribution = (24,000 × \$0.20) + (12,000 × \$0.3) = \$8,400.

Point D

Total contribution = 40,000 × \$0.20 = \$8,000.

Comparing B, C and D, we can see that contribution is maximised at C, by making 24,000 units of product A and 12,000 units of product B, to earn a contribution of \$8,400.

6 Slack and surplus 6/08

FAST FORWARD

Slack occurs when maximum availability of a resource is not used. **Surplus** occurs when more than a minimum requirement is used.

Key term

Slack occurs when maximum availability of a resource is not used.

If, at the optimal solution, the resource used equals the resource available there is **no spare capacity** of a resource and so there is **no slack.**

If a resource which has a **maximum availability** is **not binding** at the optimal solution, there will be **slack**.

For example, a machine shop makes boxes (B) and tins (T). Contribution per box is \$5 and per tin is \$7. A box requires 3 hours of machine processing time, 16kg of raw materials and 6 labour hours. A tin requires 10 hours of machine processing time, 4kg of raw materials and 6 labour hours. In a given month, 330 hours of machine processing time are available, 400kg of raw material and 240 labour hours. The manufacturing technology used means that at least 12 tins must be made every month. The constraints are:

$3B + 10T \leq 330$
$16B + 4T \leq 400$
$6B + 6T \leq 240$
$T \geq 12$

The optimal solution is found to be to manufacture 10 boxes and 30 tins.

If we substitute these values into the inequalities representing the constraints, we can determine whether the constraints are binding or whether there is slack.

Machine time:	$(3 \times 10) + (10 \times 30) = 330$ = availability Constraint is **binding**.
Raw materials:	$(16 \times 10) + (4 \times 30) = 280 < 400$ There is **slack** of 120kg of raw materials.
Labour:	$(6 \times 10) + (6 \times 30) = 240$ = availability Constraint is **binding**.

If a minimum quantity of a resource must be used and, at the optimal solution, **more than that quantity is used**, there is a **surplus** on the minimum requirement. This is shown here in the production of tins where the optimal production is 30 tins but $T \geq 12$. There is therefore a **surplus** of 18 tins over the minimum production requirement.

You can see from this that slack is associated with $\leq$ constraints and surplus with $\geq$ constraints. Machine time and labour are **binding constraints** so they have been used to their full capacity. It can be argued that if more machine time and labour could be obtained, more boxes and tins could be produced and contribution increased.

7 Shadow prices 6/08, 6/10, 12/10

FAST FORWARD

The **shadow price** or **dual price** of a limiting factor is the increase in value which would be created by having one additional unit of the limiting factor at the original cost.

Key term

The **shadow price** is the increase in contribution created by the availability of an extra unit of a limited resource at its original cost.

7.1 Limiting factors and shadow prices

Whenever there are limiting factors, there will be **opportunity costs**. As you know, these are the **benefits forgone** by using a limiting factor in one way instead of in the next most profitable way.

For example, suppose that an organisation provides two services X and Y, which earn a contribution of \$24 and \$18 per unit respectively. Service X requires 4 labour hours, and service Y 2 hours. Only 5,000 labour hours are available, and potential demand is for 1,000 of each of X and Y.

Labour hours would be a limiting factor, and with X earning \$6 per hour and Y earning \$9 per hour, the profit-maximising decision would be as follows.

	Services	Hours	*Contribution* \$
Y	1,000	2,000	18,000
X (balance)	750	3,000	18,000
		5,000	36,000

Priority is given to Y because the **opportunity cost** of providing Y instead of more of X is \$6 per hour (X's contribution per labour hour), and since Y earns \$9 per hour, the incremental benefit of providing Y instead of X would be \$3 per hour.

If extra labour hours could be made available, more X (up to 1,000) would be provided, and an extra contribution of \$6 per hour could be earned. Similarly, if fewer labour hours were available, the decision would be to provide fewer X and to keep provision of Y at 1,000, and so the loss of labour hours would cost the organisation \$6 per hour in lost contribution. This \$6 per hour, the **marginal contribution-earning potential of the limiting factor at the profit-maximising output level**, is referred to as the **shadow price** (or **dual price)** of the limiting factor.

Note that the shadow price only applies while the extra unit of resource can be obtained at its normal variable cost. The shadow price also indicates the amount by which contribution could fall if an organisation is deprived of one unit of the resource.

The shadow price of a resource is its **internal opportunity cost.** This is the marginal contribution towards fixed costs and profit that can be earned for each unit of the limiting factor that is available.

Depending on the resource in question, shadow prices enable management to make **better informed decisions** about the payment of overtime premiums, bonuses, premiums on small orders of raw materials and so on.

7.2 Linear programming and shadow prices

In terms of linear programming, the **shadow price** is the **extra contribution or profit that may be earned by relaxing by one unit a binding resource constraint**.

Suppose the availability of materials is a binding constraint. If one extra kilogram becomes available so that an alternative production mix becomes optimal, with a resulting increase over the original production mix contribution of $2, the shadow price of a kilogram of material is $2.

Note, however, that this increase in contribution of $2 per extra kilogram of material made available is calculated on the **assumption** that the **extra kilogram would cost the normal variable amount**.

Note the following points.

(a) The shadow price therefore represents the maximum **premium** above the basic rate that an organisation should be **willing to pay for one extra unit** of a resource.

(b) Since shadow prices indicate the effect of a one unit change in a constraint, they provide a measure of the **sensitivity** of the result.

(c) The **shadow price** of a constraint that is **not binding** at the optimal solution is **zero**.

(d) Shadow prices are **only valid for a small range** before the constraint becomes non-binding or different resources become critical.

7.3 Example: Calculating shadow prices

This example re-visits the scenario faced by WX Co which was used to demonstrate the graphical method of linear programming earlier in the chapter. The key points are re-capped below.

WX manufactures two products, A and B. Both products pass through two production departments, mixing and shaping. The organisation's objective is to maximise contribution to fixed costs.

Product A is sold for $1.50 whereas product B is priced at $2.00. There is unlimited demand for product A but demand for B is limited to 13,000 units per annum. The machine hours available in each department are restricted to 2,400 per annum. Other relevant data are as follows.

Machine hours required	*Mixing*	*Shaping*
	Hrs	Hrs
Product A	0.06	0.04
Product B	0.08	0.12

Variable cost per unit	$
Product A	1.30
Product B	1.70

The constraints are:

$$\begin{aligned} 0.06x + 0.08y &\leq 2{,}400 \\ 0.04x + 0.12y &\leq 2{,}400 \\ y &\leq 13{,}000 \\ x,y &\geq 0 \end{aligned}$$

The objective function is:

Contribution (C) = 0.2x + 0.3y

The availability of time in both departments are limiting factors because both are used up fully in the optimal product mix. Let us therefore calculate the effect if **one extra hour of shaping department machine time** was made available so that 2,401 hours were available.

The new optimal product mix would be at the intersection of the two constraint lines 0.06x + 0.08y = 2,400 and 0.04x + 0.12y = 2,401.

Solution by simultaneous equations gives x = 23,980 and y = 12,015.

(You should solve the problem yourself if you are doubtful about the derivation of the solution.)

Product	Units	*Contribution per unit* $	*Total contribution* $
A	23,980	0.20	4,796.0
B	12,015	0.30	3,604.5
			8,400.5
Contribution in original problem ((24,000 × $0.20) + (12,000 × $0.30))			8,400.0
Increase in contribution from one extra hour of shaping time			0.5

The **shadow price** of an hour of machining time in the shaping department is therefore the standard machine cost *plus* $0.50.

This means that extra machine capacity could be rented, for example, provided the cost premium is less than $0.50 per hour.

This value of machine time only applies as long as shaping machine time is a limiting factor. If more and more machine hours become available, there will eventually be so much machine time that it is no longer a limiting factor.

Question

Shadow prices

What is the shadow price of one hour of machine time in the mixing department?

A $3
B $7
C $10.50
D $1,193

Answer

The correct answer is A.

If we assume one more hour of machine time in the mixing department is available, the new optimal solution is at the intersection of 0.06x + 0.08y = 2,401 and 0.04x + 0.12y = 2,400

Solution by simultaneous equations gives x = 24,030, y = 11,990

Product	Units	*Contribution per unit* $	*Total contribution* $
A	24,030	0.20	4,806
B	11,990	0.30	3,597
			8,403
Contribution in original problem			8,400
Reduction in contribution			3

∴ Shadow price of one hour of machine time in the mixing department is $3.

Chapter Roundup

- All companies are limited in their capacity, either for producing goods or providing services. There is always one resource that is most restrictive (the **limiting factor**).
- In a situation where a company must **sub-contract work to make up a shortfall in its own in-house capabilities**, its total costs will be minimised if those units bought have the lowest extra variable cost of buying per unit of scarce resource saved by buying.
- **Linear programming** is a technique for solving problems of profit maximisation or cost minimisation and resource allocation. 'Programming' has nothing to do with computers: the word is simply used to denote a series of events. If a scenario contains **two or more limiting factors**, linear programming must be applied.
- **Linear programming**, at least at this fairly simple level, is a technique that can be carried out in a fairly 'handle turning' manner once you have got the basic ideas sorted out. The steps involved are as follows.

  1 Define variables
  2 Establish constraints
  3 Construct objective function
  4 Graph constraints
  5 Establish feasible region
  6 Determine optimal solution
- The **optimum solution** can be found by 'sliding the iso-contribution line out'.
- The optimal solution can also be found using **simultaneous equations**
- **Slack** occurs when maximum availability of a resource is not used. **Surplus** occurs when more than a minimum requirement is used.
- The **shadow price** or **dual price** of a limiting factor is the increase in value which would be created by having one additional unit of the limiting factor at the original cost.

Quick Quiz

1 *Fill in the blanks.*

The shadow price of a scarce resource indicates the amount by which contribution would .... if an organisation were deprived of one unit of the resource. The shadow price only applies while the extra unit of resource can be obtained at its .... cost.

2 *Put the following steps in the graphical approach to linear programming in the correct order.*

Draw a graph of the constraints	Establish constraints
Define variables	Construct objective function
Establish the feasible region	Determine optimal product mix

3 In what circumstances does slack arise?

A At the optimal solution, when the resource used equals the resource available

B At the optimal solution, when a minimum quantity of a resource must be used, and more than that quantity is used

C At the optimal solution, when the resource used is less than the resource available

D At the optimal solution, when a minimum quantity of resource is used

Answers to Quick Quiz

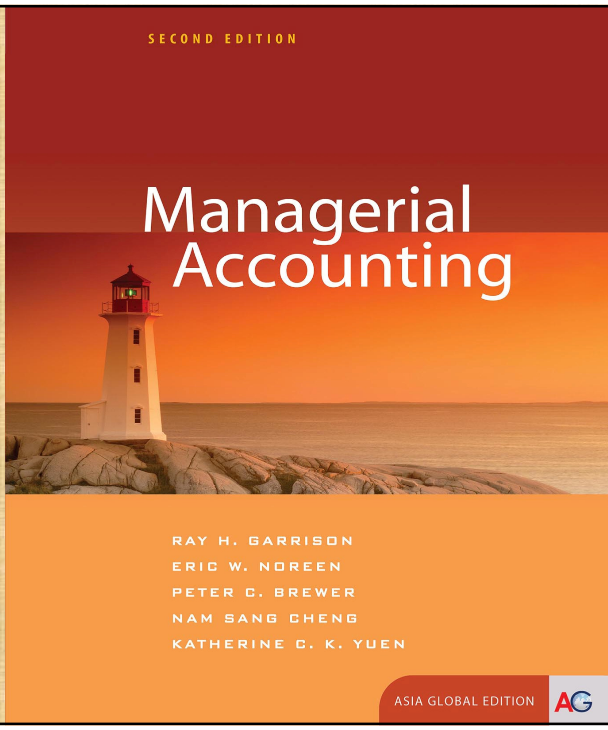
1 fall
normal variable

2 Define variables
Establish constraints
Construct objective function
Draw a graph of the constraints
Establish the feasible region
Determine optimal product mix

3 C. If a resource has a maximum availability and it's not binding at the optimal solution, there will be slack.

Now try the question below from the Exam Question Bank

Number	Level	Marks	Time
Q8	Examination	20	36 mins



Cost-Volume-Profit Relationships

Chapter 4

Learning Objective 1

Explain how changes in activity affect contribution margin and net operating income.

Basics of Cost-Volume-Profit Analysis

The contribution income statement is helpful to managers in judging the impact on profits of changes in selling price, cost, or volume. The emphasis is on cost behavior.

Racing Bicycle Company Contribution Income Statement For the Month of June		
Sales (500 bicycles)	$	250,000
Less: Variable expenses		150,000
Contribution margin		100,000
Less: Fixed expenses		80,000
Net operating income	$	20,000

Contribution Margin (CM) is the amount remaining from sales revenue after variable expenses have been deducted.

Basics of Cost-Volume-Profit Analysis

Racing Bicycle Company Contribution Income Statement For the Month of June		
Sales (500 bicycles)	$	250,000
Less: Variable expenses		150,000
Contribution margin		100,000
Less: Fixed expenses		80,000
Net operating income	$	20,000

CM is used first to cover fixed expenses. Any remaining CM contributes to net operating income.

The Contribution Approach

Sales, variable expenses, and contribution margin can also be expressed on a per unit basis. If Racing sells an additional bicycle, $200 additional CM will be generated to cover fixed expenses and profit.

Racing Bicycle Company Contribution Income Statement For the Month of June				
	Total		Per Unit	
Sales (500 bicycles)	$	250,000	$	500
Less: Variable expenses		150,000		300
Contribution margin		100,000	$	200
Less: Fixed expenses		80,000		
Net operating income	$	20,000		

The Contribution Approach

Each month, RBC must generate at least $80,000 in total contribution margin to break-even (which is the level of sales at which profit is zero).

Racing Bicycle Company Contribution Income Statement For the Month of June				
	Total		Per Unit	
Sales (500 bicycles)	$	250,000	$	500
Less: Variable expenses		150,000		300
Contribution margin		100,000	$	200
Less: Fixed expenses		80,000		
Net operating income	$	20,000		

The Contribution Approach

If RBC sells 400 units in a month, it will be operating at the *break-even point*.

Racing Bicycle Company Contribution Income Statement For the Month of June				
	Total		Per Unit	
Sales (400 bicycles)	$	200,000	$	500
Less: Variable expenses		120,000		300
Contribution margin		80,000	$	200
Less: Fixed expenses		80,000		
Net operating income	$	-		

The Contribution Approach

If RBC sells one more bike (401 bikes), net operating income will increase by $200.

Racing Bicycle Company Contribution Income Statement For the Month of June		
	Total	Per Unit
Sales (401 bicycles)	$ 200,500	$ 500
Less: Variable expenses	120,300	300
Contribution margin	80,200	$ 200
Less: Fixed expenses	80,000	
Net operating income	$ 200	

The Contribution Approach

We do not need to prepare an income statement to estimate profits at a particular sales volume. Simply multiply the number of units sold above break-even by the contribution margin per unit.

CVP Relationships in Equation Form

The contribution format income statement can be expressed in the following equation:

Profit = (Sales – Variable expenses) – Fixed expenses

Racing Bicycle Company Contribution Income Statement For the Month of June		
	Total	Per Unit
Sales (401 bicycles)	$ 200,500	$ 500
Less: Variable expenses	120,300	300
Contribution margin	80,200	$ 200
Less: Fixed expenses	80,000	
Net operating income	$ 200	

CVP Relationships in Equation Form

This equation can be used to show the profit RBC earns if it sells 401. Notice, the answer of $200 mirrors our earlier solution.

Profit = (Sales – Variable expenses) – Fixed expenses

401 units × $500

401 units × $300

$80,000

$200 = ($200,500 – $120,300) – $80,000

CVP Relationships in Equation Form

When a company has only one product we can further refine this equation as shown on this slide.

Profit = (Sales – Variable expenses) – Fixed expenses

	Quantity sold (Q)
×	**Selling price per unit (P)**
=	**Sales (Q × P)**

	Quantity sold (Q)
×	**Variable expenses per unit (V)**
=	**Variable expenses (Q × V)**

$$\text{Profit} = (P \times Q - V \times Q) - \text{Fixed expenses}$$

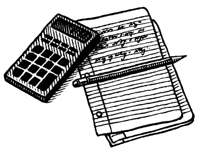
CVP Relationships in Equation Form

This equation can also be used to show the $200 profit RBC earns if it sells 401 bikes.

Profit = (Sales – Variable expenses) – Fixed expenses

$$\text{Profit} = (P \times Q - V \times Q) - \text{Fixed expenses}$$

$$\$200 = (\$500 \times 401 - \$300 \times 401) - \$80{,}000$$

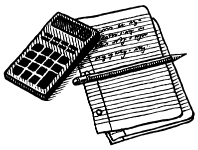


CVP Relationships in Equation Form

It is often useful to express the simple profit equation in terms of the unit contribution margin (Unit CM) as follows:

Unit CM = Selling price per unit – Variable expenses per unit
Unit CM = P – V

Profit = (P × Q – V × Q) – Fixed expenses
Profit = (P – V) × Q – Fixed expenses
Profit = Unit CM × Q – Fixed expenses

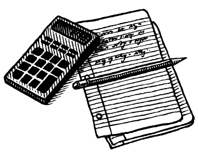


CVP Relationships in Equation Form

Profit = (P × Q – V × Q) – Fixed expenses
Profit = (P – V) × Q – Fixed expenses
Profit = Unit CM × Q – Fixed expenses

Profit = ($500 – $300) × 401 – $80,000
Profit = $200 × 401 – $80,000
Profit = $80,200 – $80,000
Profit = $200

This equation can also be used to compute RBC's $200 profit if it sells 401 bikes.



Learning Objective 2

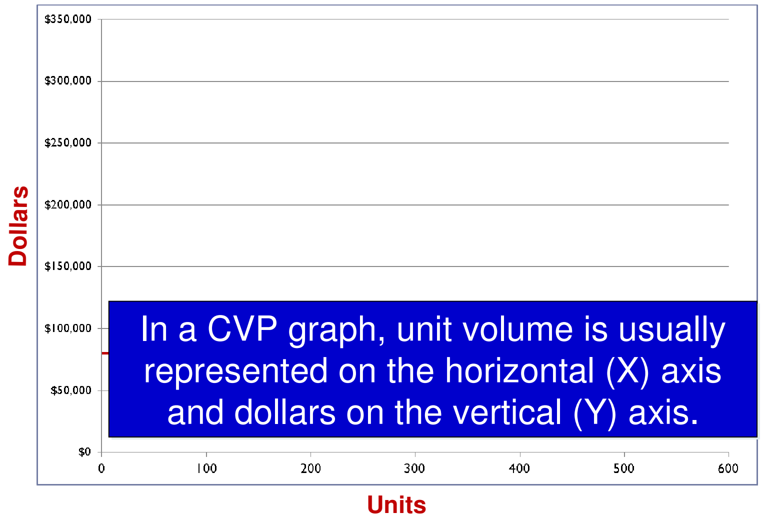
Prepare and interpret a cost-volume-profit (CVP) graph and a profit graph.

CVP Relationships in Graphic Form

The relationships among revenue, cost, profit and volume can be expressed graphically by preparing a CVP graph. Racing Bicycle developed contribution margin income statements at 0, 200, 400, and 600 units sold. We will use this information to prepare the CVP graph.

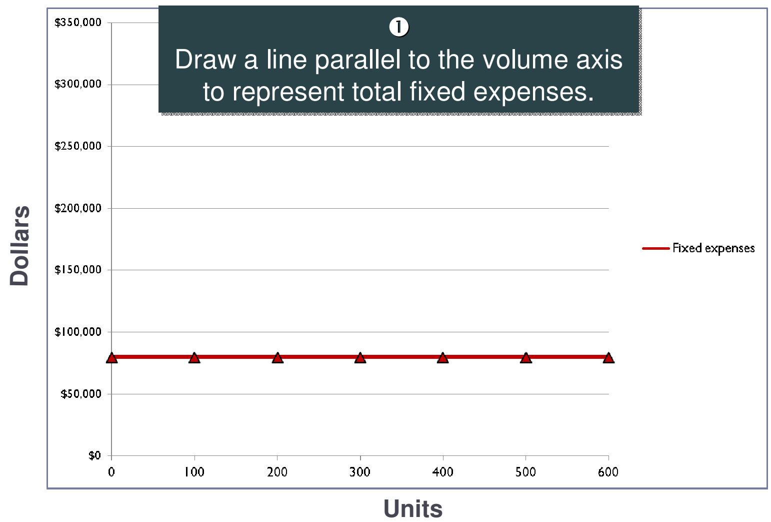
	Units Sold			
	0	200	400	600
Sales	$ -	$ 100,000	$ 200,000	$ 300,000
Total variable expenses	-	60,000	120,000	180,000
Contribution margin	-	40,000	80,000	120,000
Fixed expenses	80,000	80,000	80,000	80,000
Net operating income (loss)	$ (80,000)	$ (40,000)	$ -	$ 40,000

Preparing the CVP Graph



 Garrison, Noreen, Brewer, Cheng & Yuen

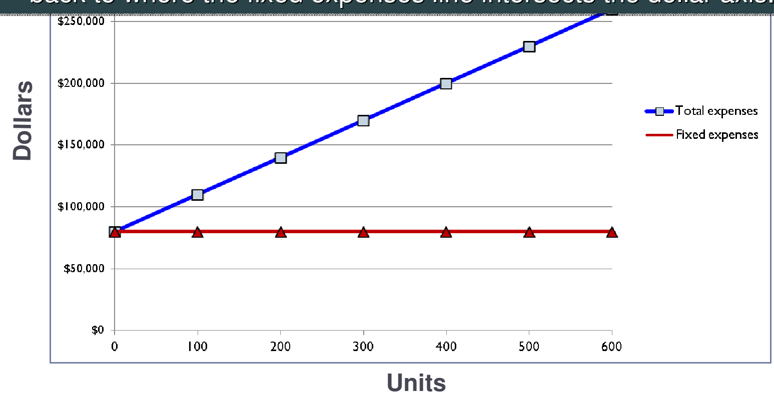
Preparing the CVP Graph



 Garrison, Noreen, Brewer, Cheng & Yuen

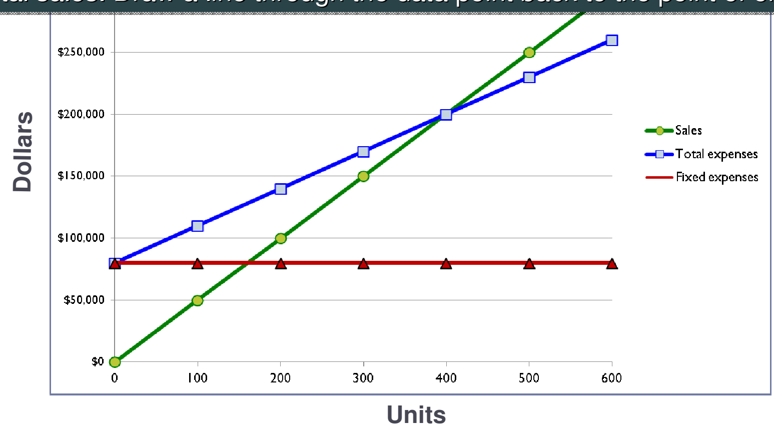
Preparing the CVP Graph

❷
Choose some sales volume, say 400 units, and plot the point representing total expenses (fixed and variable). Draw a line through the data point back to where the fixed expenses line intersects the dollar axis.

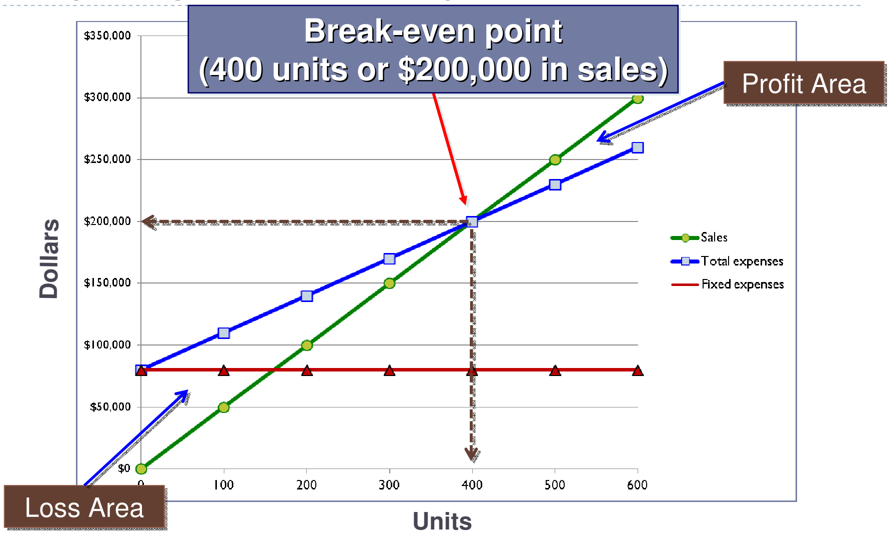


Preparing the CVP Graph

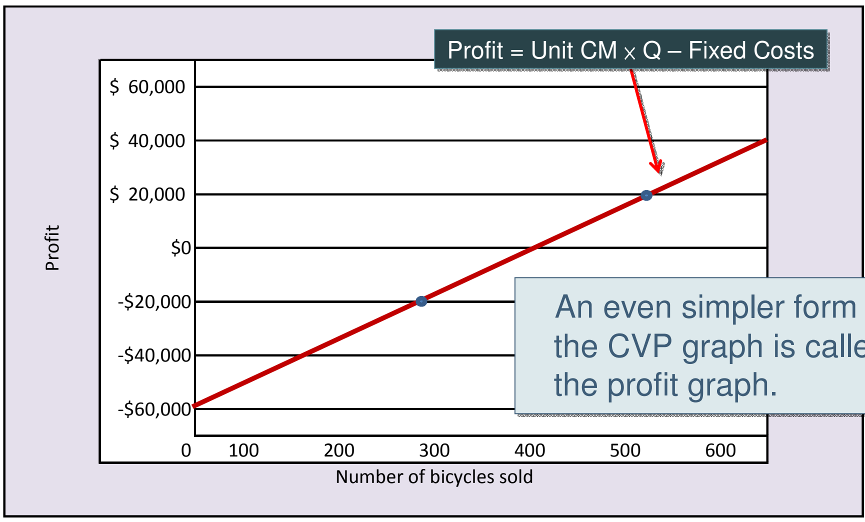
❸
Choose some sales volume, say 400 units, and plot the point representing total sales. Draw a line through the data point back to the point of origin.



Preparing the CVP Graph

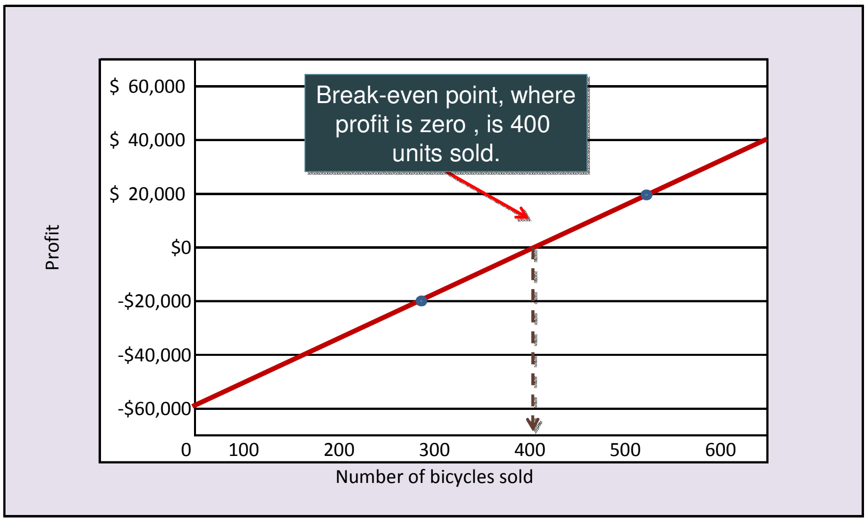


Preparing the CVP Graph



An even simpler form of the CVP graph is called the profit graph.

Preparing the CVP Graph



Learning Objective 3

Use the contribution margin ratio (CM ratio) to compute changes in contribution margin and net operating income resulting from changes in sales volume.

Contribution Margin Ratio (CM Ratio)

The CM ratio is calculated by dividing the total contribution margin by total sales.

Racing Bicycle Company
Contribution Income Statement
For the Month of June

	Total	Per Unit	CM Ratio
Sales (500 bicycles)	$ 250,000	$ 500	100%
Less: Variable expenses	150,000	300	60%
Contribution margin	100,000	$ 200	40%
Less: Fixed expenses	80,000		
Net operating income	$ 20,000		

$100,000 ÷ $250,000 = 40%

Contribution Margin Ratio (CM Ratio)

The contribution margin ratio at Racing Bicycle is:

$$\text{CM Ratio} = \frac{\text{CM per unit}}{\text{SP per unit}} = \frac{\$200}{\$500} = 40\%$$

The CM ratio can also be calculated by dividing the contribution margin per unit by the selling price per unit.

Contribution Margin Ratio (CM Ratio)

If Racing Bicycle increases sales by $50,000, contribution margin will increase by $20,000 ($50,000 × 40%).
Here is the proof:

	400 Units	500 Units
Sales	$ 200,000	$ 250,000
Less: variable expenses	120,000	150,000
Contribution margin	80,000	100,000
Less: fixed expenses	80,000	80,000
Net operating income	$ -	$ 20,000

A $50,000 increase in sales revenue results in a $20,000 increase in CM. ($50,000 × 40% = $20,000)

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the CM Ratio for Coffee Klatch?

a. 1.319
b. 0.758
c. 0.242
d. 4.139

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the CM Ratio for Coffee Klatch?

a. 1.319
b. 0.758
c. 0.242
d. 4.139

$$\text{CM Ratio} = \frac{\text{Unit contribution margin}}{\text{Unit selling price}}$$

$$= \frac{(\$1.49-\$0.36)}{\$1.49}$$

$$= \frac{\$1.13}{\$1.49} = 0.758$$

Contribution Margin Ratio (CM Ratio)

The relationship between profit and the CM ratio can be expressed using the following equation:

Profit = CM ratio × Sales – Fixed expenses

If Racing Bicycle increased its sales volume to 500 bikes, what would management expect profit or net operating income to be?

Profit = 40% × $250,000 – $80,000
Profit = $100,000 – $80,000
Profit = $20,000

Learning Objective 4

Show the effects on contribution margin of changes in variable costs, fixed costs, selling price, and volume.

The Variable Expense Ratio

The variable expense ratio is the ratio of variable expenses to sales. It can be computed by dividing the total variable expenses by the total sales, or in a single product analysis, it can be computed by dividing the variable expenses per unit by the unit selling price.

Racing Bicycle Company
Contribution Income Statement
For the Month of June

	Total	Per Unit	CM Ratio
Sales (500 bicycles)	$ 250,000	$ 500	100%
Less: Variable expenses	150,000	300	60%
Contribution margin	100,000	$ 200	40%
Less: Fixed expenses	80,000		
Net operating income	$ 20,000		

Changes in Fixed Costs and Sales Volume

What is the profit impact if Racing Bicycle can increase unit sales from 500 to 540 by increasing the monthly advertising budget by $10,000?

Changes in Fixed Costs and Sales Volume

$80,000 + $10,000 advertising = $90,000

	500 units	540 units
Sales	$ 250,000	$ 270,000
Less: Variable expenses	150,000	162,000
Contribution margin	100,000	108,000
Less: Fixed expenses	80,000	90,000
Net operating income	$ 20,000	$ 18,000

Sales *increased* by $20,000, but net operating income *decreased* by $2,000.

Changes in Fixed Costs and Sales Volume

A shortcut solution using incremental analysis

Increase in CM (40 units X $200)	$ 8,000
Increase in advertising expenses	10,000
Decrease in net operating income	$ (2,000)

Change in Variable Costs and Sales Volume

What is the profit impact if Racing Bicycle can use higher quality raw materials, thus increasing variable costs per unit by $10, to generate an increase in unit sales from 500 to 580?

Change in Variable Costs and Sales Volume

580 units × $310 variable cost/unit = $179,800

	500 units	580 units
Sales	$ 250,000	$ 290,000
Less: Variable expenses	150,000	179,800
Contribution margin	100,000	110,200
Less: Fixed expenses	80,000	80,000
Net operating income	$ 20,000	$ 30,200

Sales *increase* by $40,000, and net operating income *increases* by $10,200.

Change in Fixed Cost, Sales Price and Volume

What is the profit impact if RBC: (1) cuts its selling price by $20 per unit, (2) increases its advertising budget by $15,000 per month, and (3) increases sales from 500 to 650 units per month?

Change in Fixed Cost, Sales Price and Volume

650 units × $480 = $312,000

	500 units	650 units
Sales	$ 250,000	$ 312,000
Less: Variable expenses	150,000	195,000
Contribution margin	100,000	117,000
Less: Fixed expenses	80,000	95,000
Net operating income	$ 20,000	$ 22,000

Sales *increase* by $62,000, fixed costs increase by $15,000, and net operating income *increases* by $2,000.

Change in Variable Cost, Fixed Cost and Sales Volume

What is the profit impact if RBC: (1) pays a $15 sales commission per bike sold instead of paying salespersons flat salaries that currently total $6,000 per month, and (2) increases unit sales from 500 to 575 bikes?

Change in Variable Cost, Fixed Cost and Sales Volume

575 units x $315 = $181,125

	500 units	575 units
Sales	$ 250,000	$ 287,500
Less: Variable expenses	150,000	181,125
Contribution margin	100,000	106,375
Less: Fixed expenses	80,000	74,000
Net operating income	$ 20,000	$ 32,375

Sales *increase* by $37,500, fixed expenses *decrease* by $6,000. Net operating income *increases* by $12,375.

Change in Regular Sales Price

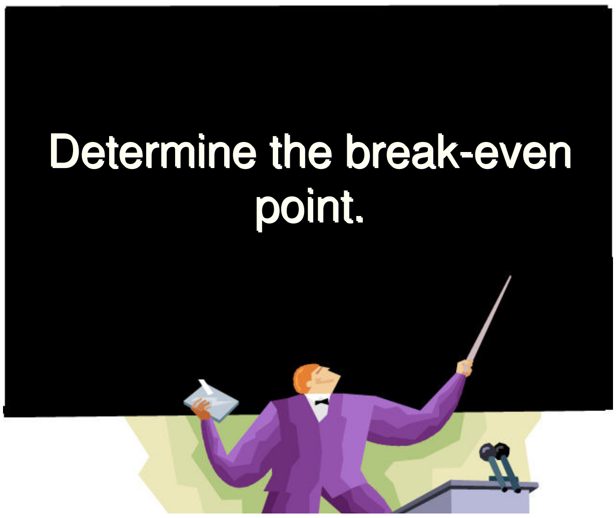
If RBC has an opportunity to sell 150 bikes to a wholesaler without disturbing sales to other customers or fixed expenses, what price would it quote to the wholesaler if it wants to increase monthly profits by $3,000?

Change in Regular Sales Price

$ 3,000 ÷ 150 bikes	=	$	20	per bike
Variable cost per bike	=		300	per bike
Selling price required	=	$	320	per bike

150 bikes × $320 per bike	=	$ 48,000
Total variable costs	=	45,000
Increase in net operating income	=	$ 3,000

Learning Objective 5



Break-even Analysis

Let's use the RBC information to complete the break-even analysis.

Racing Bicycle Company Contribution Income Statement For the Month of June			
	Total	Per Unit	CM Ratio
Sales (500 bicycles)	$ 250,000	$ 500	100%
Less: Variable expenses	150,000	300	60%
Contribution margin	100,000	$ 200	40%
Less: Fixed expenses	80,000		
Net operating income	$ 20,000		

Break-even Analysis

We can use any of the following methods to do break-even analysis:

1. Equation method
2. Formula method
3. Percentage method

Break-even in Unit Sales: Equation Method

Profits = Unit CM × Q – Fixed expenses

Suppose RBC wants to know how many bikes must be sold to break-even (earn zero profit).

$$\$0 = \$200 \times Q + \$80{,}000$$

Profits are zero at the break-even point.

Break-even in Unit Sales: Equation Method

Profits = Unit CM × Q – Fixed expenses

$$\$0 = \$200 \times Q + \$80{,}000$$

$$\$200 \times Q = \$80{,}000$$

$$Q = 400 \text{ bikes}$$

Break-even in Unit Sales: Formula Method

Let's apply the formula method to solve for the break-even point.

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{CM per unit}}$$

$$\text{Unit sales} = \frac{\$80{,}000}{\$200}$$

Unit sales = 400

Break-even in Dollar Sales: Equation Method

Suppose Racing Bicycle wants to compute the sales dollars required to break-even (zero profit). Let's use the equation method to solve this problem.

Profit = CM ratio × Sales – Fixed expenses

Solve for the unknown "Sales."

Break-even in Dollar Sales: Equation Method

Profit = CM ratio × Sales – Fixed expenses

\$ 0 = 40% × Sales – \$80,000

40% × Sales = \$80,000

Sales = \$80,000 ÷ 40%

Sales = \$200,000

Break-even in Dollar Sales: Formula Method

Now, let's use the formula method to calculate the dollar sales at the break-even point.

$$\text{Dollar sales to break even} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$\text{Dollar sales} = \frac{\$80{,}000}{40\%}$$

$$\text{Dollar sales} = \$200{,}000$$

Break-even: The Percentage Method

Now, let's use the 3rd method: the break-even percentage (BE%) method to calculate the break-even point in units as well as in sales \$. This method also efficiently calculates break-even for multiple products.

$$\text{BE\%} = \frac{\text{BE Sales \$}}{\text{Total Sales \$}} \text{ x } 100\%$$

Since $\text{BE Sales \$} = \dfrac{\text{FE}}{\text{CM\%}}$

$$\Rightarrow \text{BE\%} = \frac{\frac{\text{FE}}{\text{CM\%}}}{\text{Total Sales \$}} \text{ x } 100\%$$

Since $\text{CM\% x Total Sales \$} = \text{CM}$

$$\Rightarrow \mathbf{BE\% = \frac{FE}{CM} \text{ x } 100\%}$$

 Garrison, Noreen, Brewer, Cheng & Yuen

Break-even Units and Dollars: The Percentage Method

Applying the BE% formula to the same company RBC

$$\boxed{\mathbf{BE\% = \frac{FE}{CM} \text{ X } 100\%}}$$

$$\mathbf{BE\% = \frac{\$80{,}000}{\$100{,}000} \text{ X } 100\% = 80\%}$$

- ✓ This means that the company requires 80% of its current sales in order to break-even.
- ✓ Currently, the company's sales are \$250,000 or 500 units.
- ✓ A BE% of 80% means if the company sales are \$200,000 (\$250,000 x 80%) or 400 units (500 units x 80%), the company is break-even.
- ✓ These figures are consistent with both the equation and the formula methods.

 Garrison, Noreen, Brewer, Cheng & Yuen

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the break-even sales dollars?

a. $1,300

b. $1,715

c. $1,788

d. $3,129

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the break-even sales dollars?

a. $1,300

b. $1,715

$$\text{Break-even sales \$} = \frac{\text{Fixed expenses}}{\text{CM Ratio}} = \frac{\$1,300}{(1.49-0.36)/1.49} = \$1,715$$

$$\text{BE sales \$} = \frac{\text{Fixed Expense}}{\text{Contribution Margin}} \times \text{Current sales \$}$$

$$= \frac{\$1,300}{(\$1.49-\$0.36) \times 2,100} \times (2,100 \times \$1.49)$$

$$= \$1,715$$

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the break-even sales in units?

a. 872 cups
b. 3,611 cups
c. 1,200 cups
d. 1,150 cups

Quick Check ✓

BE sales = $\frac{\text{Fixed Expense}}{\text{Contribution Margin}}$ x Current sales

= $\frac{\$1,300}{(\$1.49-\$0.36) \text{ x } 2,100}$ x 2,100

= 1,150 cups

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the break-even sales in units?

a. 872 cups
b. 3,611 cups
c. 1,200 cups
d. 1,150 cups

Break-even = $\frac{\text{Fixed expenses}}{\text{CM per Unit}}$

= $\frac{\$1,300}{\$1.49/\text{cup} - \$0.36/\text{cup}}$

= $\frac{\$1,300}{\$1.13/\text{cup}}$

= *1,150 cups*

Learning Objective 6

Target Profit Analysis

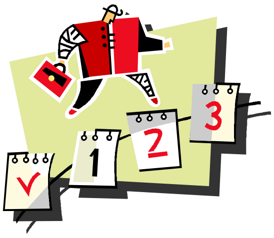
We can use any of the following methods to do target profit analysis:

1. Equation method
2. Formula method
3. Percentage method

Target Profit Analysis:
Equation Method for the Quantity required

Profit = Unit CM × Q – Fixed expenses

Our goal is to solve for the unknown "Q" which represents the quantity of units that must be sold to attain the target profit.



Target Profit Analysis:
Equation Method for the Quantity required

Suppose Racing Bicycle management wants to know how many bikes must be sold to earn a target profit of $100,000.

Profit = Unit CM × Q – Fixed expenses

$100,000 = $200 × Q – $80,000

$200 × Q = $100,000 – $80,000

Q = ($100,000 + $80,000) ÷ $200

Q = 900

Target Profit Analysis:
The Formula Method for the Quantity required

The formula uses the following equation.

$$\text{Unit sales to attain the target profit} = \frac{\text{Target profit} + \text{Fixed expenses}}{\text{CM per unit}}$$

Target Profit Analysis:
The Formula Method for the Quantity required

Suppose Racing Bicycle Company wants to know how many bikes must be sold to earn a profit of $100,000.

$$\text{Unit sales to attain the target profit} = \frac{\text{Target profit} + \text{Fixed expenses}}{\text{CM per unit}}$$

$$\text{Unit sales} = \frac{\$100{,}000 + \$80{,}000}{\$200}$$

$$\text{Unit sales} = 900$$

Target Profit Analysis:
Equation Method for the Sales $ required

Profit = CM ratio × Sales – Fixed expenses

Our goal is to solve for the unknown "Sales" which represents the dollar amount of sales that must be sold to attain the target profit.

Suppose RBC management wants to know the sales volume that must be generated to earn a target profit of $100,000.

$100,000 = 40% × Sales – $80,000
40% × Sales = $100,000 + $80,000
Sales = ($100,000 + $80,000) ÷ 40%
Sales = $450,000

Target Profit Analysis:
Formula Method for the Sales $ required

We can calculate the dollar sales needed to attain a target profit (net operating profit) of $100,000 at Racing Bicycle.

$$\textbf{Dollar sales to attain the target profit} = \frac{\textbf{Target profit + Fixed expenses}}{\textbf{CM ratio}}$$

$$\text{Dollar sales} = \frac{\$100{,}000 + \$80{,}000}{40\%}$$

Dollar sales = $450,000

Target Profit Analysis: The Percentage Method

Modifying the BE% formula to add target profit to FE

$$\text{Target Profit\%} = \frac{\text{FE+Target Profit}}{\text{CM}} \text{ x } 100\%$$

$$\text{Target Profit\%} = \frac{\$80{,}000+\$100{,}000}{\$100{,}000} \text{ x } 100\% = 180\%$$

- ✓ This means that the company requires 180% of its current sales in order to obtain the target profit.
- ✓ Currently, the company's sales are $250,000 or 500 units.
- ✓ A Target Profit % of 180% means if the company sales are $450,000 ($250,000 x 180%) or 900 units (500 units x 180%), the company has a target profit of $100,000.
- ✓ These figures are consistent with both the equation and the formula methods.

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. Use the *formula method* to determine how many cups of coffee would have to be sold to attain target profits of $2,500 per month.

a. 3,363 cups
b. 2,212 cups
c. 1,150 cups
d. 4,200 cups

Targeted sales = $\frac{\text{Fixed Expense+Targeted Profit}}{\text{Contribution Margin}}$ x Current sales

$= \frac{\$1,300+\$2,500}{(\$1.49-\$0.36) \times 2,100} \times 2,100$

= 3,363 cups

Quick

Coffee
buildin
$1.49 and the average variable expense per cup is
$0.36. Th
Use the
coffee wo
$2,500 p

a. 3,363
b. 2,212
c. 1,150
d. 4,200

$$\text{Unit sales to attain target profit} = \frac{\text{Target profit + Fixed expenses}}{\text{Unit CM}}$$

$$= \frac{\$2,500 + \$1,300}{\$1.49 - \$0.36}$$

$$= \frac{\$3,800}{\$1.13}$$

= 3,363 cups

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. Use the *formula method* to determine the sales dollars that must be generated to attain target profits of $2,500 per month.

a. $2,550
b. $5,011
c. $8,458
d. $10,555

Targeted sales \$ = $\frac{\text{Fixed Expense+Targeted Profit}}{\text{Contribution Margin}}$ x Current sales \$

$$= \frac{\$1{,}300 + \$2{,}500}{(\$1.49 - \$0.36) \times 2{,}100} \times (2{,}100 \times \$1.49)$$

= \$5,011

Quic

Coffe
build
\$1.49 and the average variable expense per cup is \$0.36. The average fixed expense per month is \$1,300.
Use the *fo*
that must
per month

a. \$2,550
b. \$5,011
c. \$8,458
d. \$10,555

Sales \$ to attain target profit = $\frac{\textbf{Target profit + Fixed expenses}}{\textbf{CM ratio}}$

$$= \frac{\$2{,}500 + \$1{,}300}{(\$1.49 - 0.36) \div \$1.49}$$

$$= \frac{\$3{,}800}{0.758}$$

= \$5,011

Learning Objective 7

Compute the margin of safety and explain its significance.

The Margin of Safety in Dollars

The margin of safety in dollars is the excess of budgeted (or actual) sales over the break-even volume of sales.

Margin of safety in dollars = Total sales - Break-even sales

Let's look at Racing Bicycle Company and determine the margin of safety.

The Margin of Safety in Dollars

If we assume that RBC has actual sales of $250,000, given that we have already determined the break-even sales to be $200,000, the margin of safety is $50,000 as shown.

	Break-even sales 400 units	Actual sales 500 units
Sales	$ 200,000	$ 250,000
Less: variable expenses	120,000	150,000
Contribution margin	80,000	100,000
Less: fixed expenses	80,000	80,000
Net operating income	$ -	$ 20,000

The Margin of Safety Percentage

RBC's margin of safety can be expressed as **20%** of sales.
($50,000 ÷ $250,000)

	Break-even sales 400 units	Actual sales 500 units
Sales	$ 200,000	$ 250,000
Less: variable expenses	120,000	150,000
Contribution margin	80,000	100,000
Less: fixed expenses	80,000	80,000
Net operating income	$ -	$ 20,000

The Margin of Safety

The margin of safety can be expressed in terms of the number of units sold. The margin of safety at RBC is $50,000, and each bike sells for $500; hence, RBC's margin of safety is 100 bikes.

$$\text{Margin of Safety in units} = \frac{\$50,000}{\$500} = 100 \text{ bikes}$$

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the margin of safety expressed in cups?

a. 3,250 cups
b. 950 cups
c. 1,150 cups
d. 2,100 cups

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the margin of safety expressed in cups?

a. 3,250 cups
b. 950 cups
c. 1,1
d. 2,1

Margin of safety = Total sales – Break-even sales
= 2,100 cups – 1,150 cups
= *950 cups*

Linking Margin of Safety % (to sales) and Break-even % (to sales)

Margin of safety in dollars = Total sales - Break-even sales

$$\frac{\text{Margin of safety in dollars}}{\text{Total sales in dollars}} = \text{Margin of safety percentage (MoS\%)}$$

$$= \frac{\text{Total sales} - \text{Breakeven sales}}{\text{Total sales}}$$

$$= 1 - \frac{\text{Breakeven in dollars}}{\text{Total sales in dollars}}$$

$$= 1 - \text{Breakeven percentage}$$

$$= 1 - \text{BE\%}$$

Therefore: $\text{BE\%} = 1 - \text{MoS \%}$

Breakeven Calculation

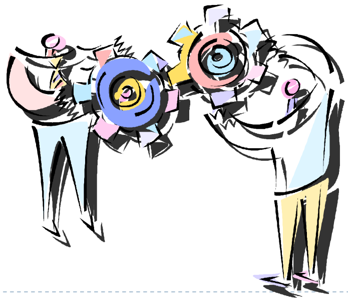
RBC's margin of safety = **20%** of sales

	Actual sales 500 units
Sales	**$ 250,000**
Less: variable expenses	**150,000**
Contribution margin	**100,000**
Less: fixed expenses	**80,000**
Net operating income	**$ 20,000**

Break-even sales of RBC = 1 – 20% = 80% of sales
= $250,000 x 80%
= $200,000
= Break-even Sales on slide 75

Cost Structure and Profit Stability

Cost structure refers to the relative proportion of fixed and variable costs in an organization. Managers often have some latitude in determining their organization's cost structure.



Cost Structure and Profit Stability

There are advantages and disadvantages to high fixed cost (or low variable cost) and low fixed cost (or high variable cost) structures.

An advantage of a high fixed cost structure is that income will be higher in good years compared to companies with lower proportion of fixed costs.

A disadvantage of a high fixed cost structure is that income will be lower in bad years compared to companies with lower proportion of fixed costs.

Companies with low fixed cost structures enjoy greater stability in income across good and bad years.

Learning Objective 8

Compute the degree of operating leverage at a particular level of sales and explain how it can be used to predict changes in net operating income.

Operating Leverage

Operating leverage is a measure of how sensitive net operating income is to percentage changes in sales. It is a measure, at any given level of sales, of how a percentage change in sales volume will affect profits.

$$\text{DOL} = \text{Degree of Operating Leverage} = \frac{\text{Contribution Margin}}{\text{Net Operating Income}^{**}}$$

** Profit Before Tax is a commonly used alternative to Net Operating Income in the degree of operating leverage calculation

Operating Leverage

To illustrate, let's revisit the contribution income statement for RBC.

	Actual sales 500 Bikes
Sales	$ 250,000
Less: variable expenses	150,000
Contribution margin	100,000
Less: fixed expenses	80,000
Net income	$ 20,000

$$\text{Degree of Operating Leverage} = \frac{\$100{,}000}{\$20{,}000} = 5$$

Operating Leverage

With an operating leverage of 5, if RBC increases its sales by 10%, net operating income would increase by 50%.

Percent increase in sales		10%
Degree of operating leverage	×	5
Percent increase in profits		50%

Here's the verification!

Operating Leverage

	Actual sales (500)	Increased sales (550)
Sales	$ 250,000	$ 275,000
Less variable expenses	150,000	165,000
Contribution margin	100,000	110,000
Less fixed expenses	80,000	80,000
Net operating income	$ 20,000	$ 30,000

10% increase in sales from $250,000 to $275,000 . . .

. . . results in a 50% increase in income from $20,000 to $30,000.

Quick Check ✓

Coffee Klatch is an espresso stand in a downtown office building. The average selling price of a cup of coffee is $1.49 and the average variable expense per cup is $0.36. The average fixed expense per month is $1,300. 2,100 cups are sold each month on average. What is the operating leverage?

a. 2.21
b. 0.45
c. 0.34
d. 2.92

Quick Check ✓

Coffee Klatch is an es
downtown office buildi
price of a cup of coffe
variable expense per
fixed expense per mo
are sold each month c
operating leverage?

	Actual sales 2,100 cups
Sales	$ 3,129
Less: Variable expenses	756
Contribution margin	2,373
Less: Fixed expenses	1,300
Net operating income	$ 1,073

a. 2.21
b. 0.45
c. 0.34
d. 2.92

$$\text{Operating leverage} = \frac{\text{Contribution margin}}{\text{Net operating income}}$$

$$= \frac{\$2,373}{\$1,073} = 2.21$$

Quick Check ✓

At Coffee Klatch the average selling price of a cup of coffee is $1.49, the average variable expense per cup is $0.36, the average fixed expense per month is $1,300 and an average of 2,100 cups are sold each month.

If sales increase by 20%, by how much should net operating income increase?

a. 30.0%
b. 20.0%
c. 22.1%
d. 44.2%

Quick Check ✓

At Coffee Klatch the average selling price of a cup of coffee is $1.49, the average variable expense per cup is $0.36, the average fixed expense per month is $1,300 and an average of 2,100 cups are sold each month.

If sales increase by 20%, by how much should net operating income increase?

a. 30.0%
b. 20.0%
c. 22.1%
d. 44.2%

	Percent increase in sales	20.0%
×	Degree of operating leverage	2.21
	Percent increase in profit	44.20%

Verify Increase in Profit

	Actual sales	*Increased sales*
	2,100 cups	*2,520 cups*
Sales	$ 3,129	$ 3,755
Less: Variable expenses	756	907
Contribution margin	2,373	2,848
Less: Fixed expenses	1,300	1,300
Net operating income	$ 1,073	$ 1,548
% change in sales		20.0%
% change in net operating income		44.2%

What does higher value of Operating Leverage mean?

- High Operating Leverage ratio
  - signals the existence of high fixed costs.
  - increases risk of making loss in adverse market conditions.
  - increases opportunity to make profit when higher demand exists.
  - has lower margin of safety percentage (MoS%)

$$DOL \approx \frac{1}{MoS\%}$$

Proof of Operating Leverage and Profit Movement Relationship

	Benchmark Co.	High F.C. Co.
Total Sales **(Same)**	$3,200,000	$3,200,000
Unit selling price **(Same)**	$800	$800
Unit variable costs	($300)	($150)
Unit Contribution margin	$500	$650
Unit sales **(Same)**	4,000	4,000
Contribution margin (CM)	$2,000,000	$2,600,000
Fixed costs	($1,500,000)	($2,100,000)
Net Operating Profit **(Same)** (P)	500,000	500,000
Degree of operating leverage (CM/P)	**4.0**	**5.2**

Proof of Operating Leverage and Profit Movement Relationship

	Benchmark Co.	High F.C. Co.
Increase in sales	12.5%	12.5%
Degree of operating leverage	X 4.0	X 5.2
Increase in profits	**50%**	**65%**
Proof:		
Unit contribution margin	$500	$650
Unit change in sales (4,000 x 12.5%)	x 500	x 500
Change in profits	$250,000	$325,000
Percentage increase from the original $500,000 profit	**50%**	**65%**

Proof of Operating Leverage and MoS% Relationship

		Benchmark Co.	High F.C. Co.
Total Sales **(Same)**	(S)	$3,200,000	$3,200,000
Contribution margin	(CM)	$2,000,000	$2,600,000
Fixed costs	(F)	($1,500,000)	($2,100,000)
Net Operating Profit **(Same)**	(P)	500,000	500,000
Degree of operating leverage (CM/P)		**4.0**	**5.2**
Break-even Sales Dollars [F/(CM/S)]		2,400,000	2,584,615
Break-even % (to sales)		75%	80.77%
MoS% = 1 – BE% (see slide 77)		**25%**	**19.23%**
1/MoS% = Degree of operating leverage		**4.0**	**5.2**

Structuring Sales Commissions

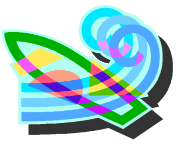
Companies generally compensate salespeople by paying them either a commission based on sales or a salary plus a sales commission. Commissions based on sales dollars can lead to *lower profits* in a company.

Let's look at an example.

 Garrison, Noreen, Brewer, Cheng & Yuen

Structuring Sales Commissions

Pipeline Unlimited produces two types of surfboards, the XR7 and the Turbo. The XR7 sells for $100 and generates a contribution margin per unit of $25. The Turbo sells for $150 and earns a contribution margin per unit of $18.



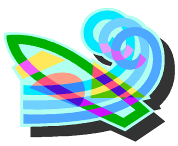
The sales force at Pipeline Unlimited is compensated based on sales commissions.

 Garrison, Noreen, Brewer, Cheng & Yuen

Structuring Sales Commissions

If you were on the sales force at Pipeline, you would push hard to sell the Turbo even though the XR7 earns a higher contribution margin per unit.

To eliminate this type of conflict, commissions can be based on contribution margin rather than on selling price alone.



Learning Objective 9

Compute the break-even point for a multiproduct company and explain the effects of shifts in the sales mix on contribution margin and the break-even point.

The Concept of Sales Mix

- Sales mix is the relative proportion in which a company's products are sold.
- Different products have different selling prices, cost structures, and contribution margins.
- When a company sells more than one product, break-even analysis becomes more complex as the following example illustrates.

Let's assume Racing Bicycle Company sells bikes and carts and that the sales mix between the two products remains the same.

Multi-Product Breakeven Analysis (The BE% Method)

RBC's Bikes and Carts sales and profit data are as follows:

	Bicycle	Carts	Total
Sales	$ 250,000	$ 300,000	$ 550,000
Variable expenses	150,000	135,000	285,000
Contribution margin	100,000	165,000	265,000
Fixed expenses			170,000
Net operating income			$ 95,000

Sales	$ 250,000	$300,000
	x BE% = 64.15%	
Breakeven sales	$160,375	$192,450

Total break-even sales = $352,825

$$\text{DOL} = \frac{\text{Contribution Margin}}{\text{Net Operating Income}} = \frac{1}{\text{MoS\%}}$$

$$\Rightarrow \text{MoS\%} = \frac{\text{Net Operating Income}}{\text{Contribution Margin}}$$

$$\Rightarrow \text{MoS\%} = \frac{95{,}000}{265{,}000} = 35.85\%$$

$$\because \text{MoS\%} = 1 - \text{BE\%}$$

$$\therefore \text{BE\%} = 1 - \text{MoS\%} = 1 - 35.85 = 64.15\%$$

Multi-Product Breakeven Analysis
(The BE% Method)

	Bicycle		Carts		Total	
Sales	$ 160,375	100%	$ 192,450	100%	$ 352,825	100.0%
Variable expenses	96,225	60%	86,603	45%	182,828	51.8%
Contribution margin	64,150	40%	105,847	55%	169,997	48.2%
Fixed expenses					170,000	
Net operating income			Rounding error		$ (3)	

Multi-Product Breakeven Analysis
(The CM Ratio Method)

Bikes comprise 45% of RBC's total sales revenue and the carts comprise the remaining 55%. RBC provides the following information:

	Bicycle		Carts		Total	
Sales	$ 250,000	100%	$ 300,000	100%	$ 550,000	100.0%
Variable expenses	150,000	60%	135,000	45%	285,000	51.8%
Contribution margin	100,000	40.0%	165,000	55%	265,000	48.2%
Fixed expenses					170,000	
Net operating income					$ 95,000	
Sales mix	$ 250,000	45%	$ 300,000	55%	$ 550,000	100%

$$\frac{\$265,000}{\$550,000} = 48.2\% \text{ (rounded)}$$

Multi-Product Breakeven Analysis (The CM Ratio Method)

Dollar sales to break even = **Fixed expenses / CM ratio**

Dollar sales to break even = **$170,000 / 48.2%** = **$352,697**

	Bicycle		Carts		Total	
Sales	$ 158,714	100%	$ 193,983	100%	$ 352,697	100.0%
Variable expenses	95,228	60%	87,293	45%	182,521	51.8%
Contribution margin	63,485	40%	106,691	55%	170,176	48.2%
Fixed expenses					170,000	
Net operating income			Rounding error		$ 176	
Sales Mix	$ 158,714	45%	$ 193,983	55%	$ 352,697	100.0%

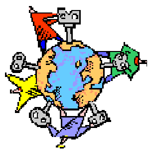
Compare the Breakeven Results calculated by the BE% and CM ratio methods

	Bicycle		Carts		Total		Rounding difference from the breakeven
Breakeven Sales Mix							
The BE% method	$ 160,375	45%	$ 192,450	55%	$ 352,825	100%	$ 3
The CM ratio method	$ 158,714	45%	$ 193,983	55%	$ 352,697	100%	$ 176

Using different methods to calculate the break-even points will result in slightly different answers due to rounding differences at different points of the calculations. In this example, the BE% seems to provide a better estimation.

Key Assumptions of CVP Analysis

1. Selling price is constant.
2. Costs are linear and can be accurately divided into variable (constant per unit) and fixed (constant in total) elements.
3. In multiproduct companies, the sales mix is constant.
4. In manufacturing companies, inventories do not change (units produced = units sold).



End of Chapter 4